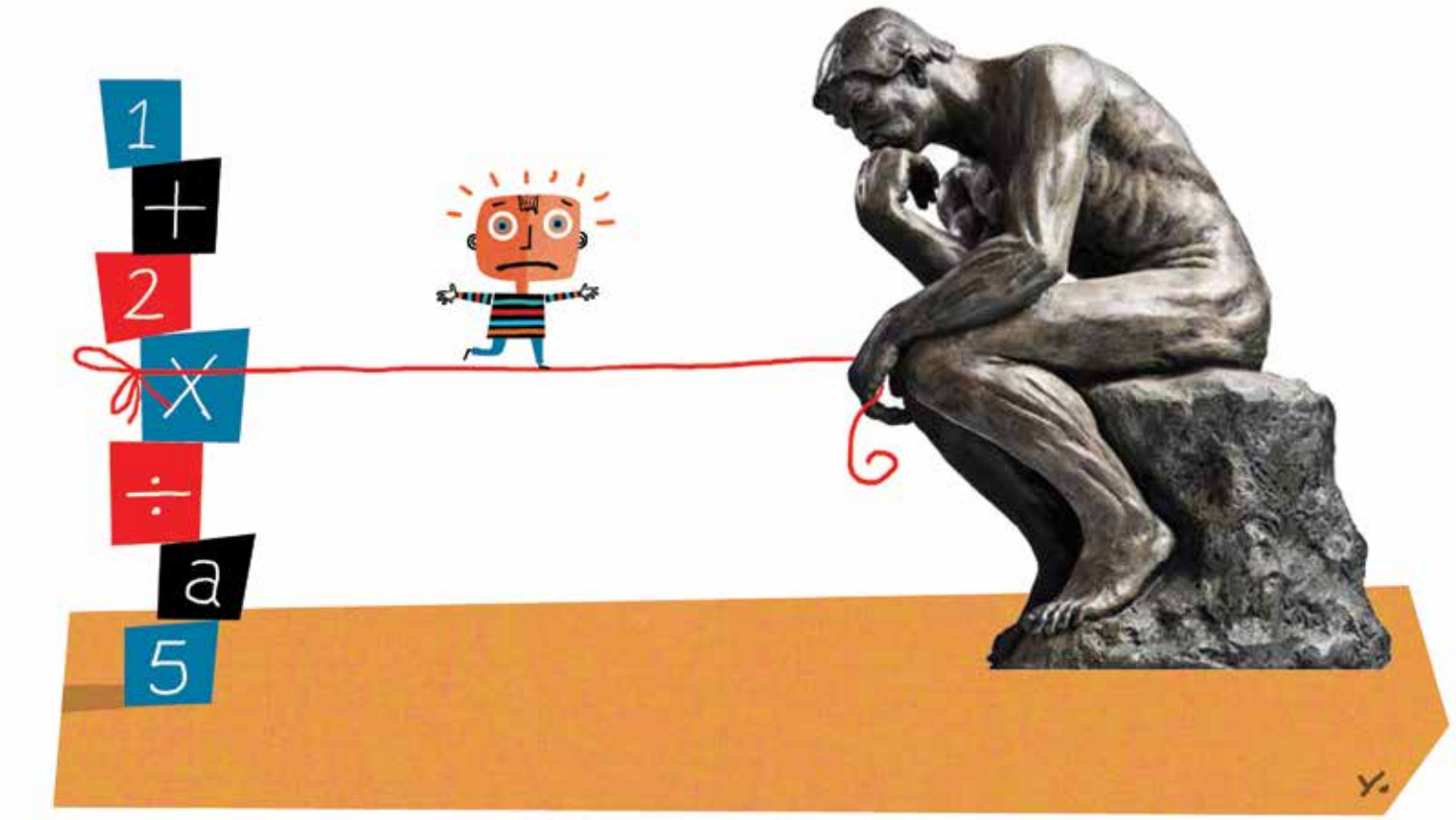
ASK THE COGNITIVE SCIENTIST

Math Anxiety: Can Teachers Help Students Reduce It?



How does the mind work—and especially how does it learn? Teachers' instructional decisions are based on a mix of theories learned in teacher education, trial and error, craft knowledge, and gut instinct. Such knowledge often serves us well, but is there anything sturdier to rely on?

Cognitive science is an interdisciplinary field of researchers from psychology, neuroscience, linguistics, philosophy, computer science, and anthropology who seek to understand the mind. In this regular American Educator *column, we consider findings from this field that are strong and clear enough to merit classroom application.*

By Sian L. Beilock and Daniel T. Willingham

Question: Some of my students seem to get really nervous about math. I can understand not liking the subject very much—to be honest, I don't love it myself—but their nervousness seems to get in the way of their understanding. How can I reassure them or otherwise make them less anxious?

Answer: There is no doubt that math makes some students very anxious. This problem can begin as early as elementary school, and might be prompted both by genuine concerns—the student perceives that his or her math skills need work—and by social cues that subtly convey the message that math should be feared. Research on how to best help students through this problem is ongoing, but there are a few techniques that teachers may find useful.

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ILLUSTRATIONS BY JAMES YANG

To many people, "math" is a scary four-letter word. They don't like it, they don't feel like they are very good at it, and they just want to stay away from it. People who feel tension, apprehension, and fear of situations involving math are said to have *math anxiety*. And, perhaps not surprisingly, math anxiety is associated with poor math performance in school. Students with a high degree of math anxiety perform worse in math from elementary school through college, relative to their less math-anxious counterparts.[1] But, it's not just school situations where a negative relationship between math anxiety and mathematical performance emerges. Higher levels of math anxiety are associated with poor calculations of drug dosages by nurses and impaired financial planning.[2]

Math anxiety is not limited to a minority of individuals nor to one country. International comparisons of high school students show that some students in every country are anxious about math. It is perhaps unsurprising that there is an inverse relationship between anxiety and efficacy: countries where kids are less proficient in math (as measured by the Program for International Student Assessment, or PISA) tend to have higher levels of math anxiety.[3] In the United States, an estimated 25 percent of four-year college students and up to 80 percent of community college students suffer from a moderate to high degree of math anxiety.[4] Most students report having at least one negative experience with math at some point during their schooling.[5]

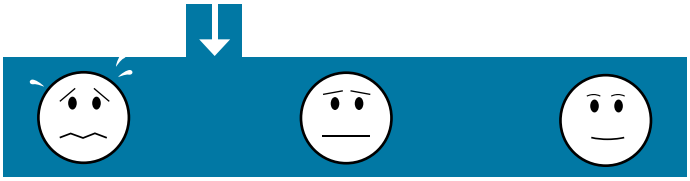
Anecdotally, most of us can recall a time when we overheard a friend, colleague, or family member talk about his dislike for math or how she is "not a numbers person." This is a notable contrast to reading; few people cheerfully volunteer that they just aren't very good readers. It seems socially acceptable to be anxious about math.

Because math anxiety is widespread and often tied to poor math skills, it's imperative to understand when anxiety about math starts to emerge, where it comes from, and what we can do to alleviate it. Only then can we start to attack the phenomenon, identifying strategies that target both how material is taught and how students feel about math, as a means to lower math anxiety, raise math achievement, and ensure that we are equipping students with the level of mathematics knowledge needed for the 21st-century workplace. Although research on math anxiety goes back to the 1970s, it has really gained momentum only in the last 10 years or so. Still, in that time, we have learned much about its origins and some ways to combat it.

When and How Does Math Anxiety Emerge?

Recently, several studies have examined early elementary students, and they indicate that math anxiety starts early. Although the specific details of these studies vary, the general questions have been similar: Do early elementary students report math anxiety and, if so, how is it related to math performance?

In one recent study, math anxiety was assessed in 154 first- and second-graders with a newly developed scale that asked them questions like, "How do you feel when taking a big test in your math class?" or "How do you feel when getting your math book and seeing all the numbers in it?"[6] Kids responded by using a sliding scale that featured a calm face on the far right, a moderately nervous face in the middle, and an obviously nervous face on the far left (see figure below).



SOURCE: GERARDO RAMIREZ, ELIZABETH A. GUNDERSON, SUSAN C. LEVINE, AND SIAN L. BEILOCK, "MATH ANXIETY, WORKING MEMORY, AND MATH ACHIEVEMENT IN EARLY ELEMENTARY SCHOOL," *JOURNAL OF COGNITION AND DEVELOPMENT* 14 (2013): 187–202. REPRINTED BY PERMISSION OF TAYLOR & FRANCIS LTD, WWW.TANDF.CO.UK/JOURNALS.

Several days later, they completed a standardized test of math achievement (the Woodcock-Johnson III Applied Problems subtest).[7] The test included items like identifying the correct time on a clock, money calculations, and word problems requiring arithmetic or simple fraction work.

Do first- and second-graders report having math anxiety? Yes. Averaging across all the questions, nearly 50 percent of the students reported medium to high levels of math anxiety, being "moderately nervous" to "very, very nervous" about math. Do these reports of math anxiety relate to students' math achievement? Yes, and in the way you would expect: higher math anxiety was associated with lower achievement (though, as we discuss below, this relation was stronger for some students than others). Finally, math anxiety's predictive power was specific to math—there was little association between math anxiety and performance on a reading comprehension test.

Because math anxiety is widespread and tied to poor math skills, we must understand what we can do to alleviate it.

Is Math Anxiety Just Another Name for "Bad at Math"?

We've just reviewed findings that math anxiety and math achievement are related. But how could it be otherwise? After all, math-anxious individuals stay away from math courses and math-related situations, and they learn less math in the courses they do take. Indeed, it's tempting to conclude that their anxiety is logical—they are anxious because they are bad at math. For that matter, maybe the whole notion of "math anxiety" is not useful. Some might assume it's pretty much just another name for "poor math skills."

Math anxiety implies more than "bad at math." It implies that someone would be better at math if he or she weren't so anxious. And there is evidence that's true. A growing body of work shows that math anxiety robs people of working memory. You can think of working memory as a kind of mental scratch pad—it's what allows you to keep several things in mind simultaneously, and to

manipulate them in order to think and solve problems. For example, suppose a parent says to a teenager, "Your chores this afternoon are to clean the cat litter box, set the table for dinner, and take out the trash. And if you could chop some vegetables for the stew I'm going to make later, that would be nice." The teen thinks, "Chopping vegetables and cleaning the cat box will make a mess, so I should take out the trash after I do those chores. And my hands should be clean when I set the table and when I chop vegetables. So I guess I'll wash my hands, then set the table, then chop vegetables, then clean the cat box, then take out the trash." Working memory is needed to keep the four chores in mind *and* to think about the consequences of doing each one in a particular sequence *and* to construct that sequence.

As you can imagine, if our teen had been given 10 chores instead of four, she would not have been able to keep them all in mind. Working memory can only hold so much. And the amount of "space" in working memory varies from person to person. Given that working memory is important for solving problems, it's not surprising that one's working memory capacity is related to one's problem-solving and reasoning ability and to measures of general intelligence.[8]

The role of working memory in thinking helps us understand the destructive consequences of math anxiety; anxious thoughts consume valuable working memory space.[9] Math anxiety essentially prompts students to do two things at once: solve the math problem *and* deal with worries about the math (including worries about getting the problem wrong, looking foolish, and what others may think of them). As a result, they have less working memory to devote to the math, and their math performance suffers.

Neuroscientific data also support this interpretation. For example, one group of researchers explored neural activity in brain areas associated with negative emotions and in brain areas known to support numerical computations while third-grade children—both those lower and those higher in math anxiety—performed math problems.[10] When performing mathematical calculations, math-anxious children, relative to their less anxious peers, show more brain activity in the right amygdala (known to be important for processing negative emotions). This increased amygdala activity was accompanied by a reduction in activity in brain regions known to support working memory and numerical processing (e.g., the dorsolateral prefrontal cortex and the posterior parietal lobe). Using similar functional magnetic resonance imaging (fMRI) methods, another group of researchers found that the higher one's math anxiety, the larger the increase in activity in brain regions associated with threat and the experience of pain.[11] Interestingly, we observed this relation when highly math-anxious people just *anticipated* doing math.

Which Students Are Most Susceptible?

Math anxiety may start when children are quite young, but it can't come out of nowhere. What prompts it? Factors related to both students' math abilities at the start of elementary school and students' social environment (in the classroom, at home, and in society in general) likely play a role in the development of math anxiety.

We know that adults with math anxiety tend to have shortfalls in one or more of the basic building blocks of mathematical thinking and reasoning. These building blocks include skills like counting objects, deciding which of two numbers represents the larger quantity, and mentally rotating three-dimensional objects.[12] We have speculated that a poor grasp of basic math building blocks early in schooling may predispose students to develop math anxiety, partly in response to their potential struggles in math. It seems predictable that students who struggle with math would be more likely to become anxious about it.

Math anxiety robs people of working memory, which is important for solving problems.

Another characteristic of kids is important, but this one doesn't predict who is likely to suffer from anxiety. Instead, it predicts whose math performance is most disrupted should they get anxious. And the finding is rather counter-intuitive: kids with the highest level of working memory show the most pronounced negative relation between math anxiety and math achievement.[13] In other words, students with the most cognitive horsepower seem to suffer the most as a function of math anxiety. How can this be? Math anxiety depresses math performance because it eats up working memory space. Wouldn't these students have spare working memory capacity, so anxiety would have less of an impact?

The answer to this question is not completely clear, but one possibility is that students with the most working memory tend to rely on more advanced problem-solving strategies;[14] presumably, they're in the habit of using these cognitively demanding strategies because they typically have the mental resources to carry them out. For instance, a simple strategy for a first-grader solving the problem "8 + 4 = ?" would be counting on his fingers. A strategy that demands more of working memory would be

decomposition, or breaking down units so that they are easier to process (e.g., 8 + 4 → 8 + 2 + 2). Because the advanced strategies demand more working memory, they are more sensitive to anxiety's deleterious effects. Ironically, something that usually helps kids in math—large working memory capacity—becomes vulnerable to disruption when they are anxious.

Social Influences and Math Anxiety

There is some evidence that children might pick up on cues from parents, teachers, or peers that math is, indeed, worthy of anxiety. Children who start schooling with deficiencies in basic mathematical skills may be especially predisposed to pick up on social cues (e.g., their teachers' behavior) that highlight math in negative terms.[15]

There is also evidence of a more general link between teachers' behavior and students' math performance. In a preliminary study of 17 teachers and 117 first- and second-grade students, researchers found that female elementary school teachers' math anxiety (over 90 percent of elementary school teachers in the United States are female) related to their female students' math achievement at the end of the school year—the higher a teacher's math anxiety, the lower her female students' math achievement by the end of the school year (that's after accounting for girls' beginning-of-the-year math achievement and teachers' math knowledge).[16] Initially, we interpreted our findings as being specific to girls (a transmission of math negativity from female teachers to female students). However, in a large-scale follow-up enlisting more than 70 teachers and 650 of their first- and second-grade students, we found that teachers' math anxiety also is negatively related to boys' math achievement (albeit not as strongly) at the end of the school year. Regardless of a student's gender, his or her teacher's math anxiety seems to carry implications for the student's level of math achievement.[17]

Of course, there are many sources from which negativity about math could develop—ranging from parents to the media. But, clearly, information about positive and negative aspects of math can be found in the classroom, and it seems, at least at first glance, that not only do kids pick up on this negativity but it also carries implications for their math achievement across the school year.

What Can Teachers Do about Math Anxiety?

While there is still a lot of work to be done to gain a complete understanding of math anxiety, knowing something about where math anxiety comes from, how it relates to math performance, and whom it is most likely to affect helps us start to think about the remediation of math anxiety.

Ensure fundamental skills. Enhancing basic numerical and spatial processing may help guard against the development of math anxiety in young students. Research shows that the quality of numerical and spatial talk by parents in the home is related to children's math and spatial skills.[18] Thus, something as simple as encouraging parents to engage with young children around math may help ensure that children come to school with basic mathematical competencies that help prevent math anxiety. On the flip side, identification of at-risk students, coupled with targeted exercises designed to boost their basic mathematical competencies and regulate their potential anxieties, may help to prevent at-risk children from developing math anxiety.

A course on *how* to teach math concepts seems to be more effective in addressing math anxiety among pre-service teachers than a course on math concepts themselves.

Focus on teacher training. Knowledge that a teacher's math anxiety can affect her students' math achievement suggests that we also need to ensure that teachers feel fully confident in their preparation to teach math. Researchers have found that a course focused on *how* to teach math concepts was more effective in addressing math anxiety among pre-service teachers than a course focused directly on the math concepts themselves.[19] This point is especially salient with the onset of new curricula prompted by the Common Core State Standards. Even experienced teachers may be asked to teach new material.

Try reducing anxiety by changing the assessment. Math anxiety depresses math performance because it occupies working memory. Research has shown that math anxiety is more strongly linked to poor performance when students take a timed test.[20] There are likely several reasons why alleviating time pressure makes math anxiety less of a problem, from reducing worries about not finishing in time, to giving students the time and space to work through their answers.

Try reducing anxiety through a writing exercise. Giving students the opportunity to write freely about their emotions for about 10 minutes with respect to a specific situation (e.g., an

upcoming exam) can help boost test performance. Writing is thought to alleviate the burden that negative thoughts place on working memory by affording people an opportunity to re-evaluate the stressful experience, such as thinking, "Oh, maybe this math test isn't really *that* big of a deal." In recent work, we showed that writing before an upcoming math test helped reduce the performance gap between students with higher levels of math anxiety and those with lower levels,[21] and others have shown that this writing exercise can be beneficial for test taking in general, whether it is the MCAT[22] or a high school biology final.[23] Of course, such writing may not be appropriate for young students, which means there is still more work to be done to determine how to alleviate the math anxiety that some students feel at the start of formal schooling.

Below is an example of how we have prompted students to put their thoughts down in writing before an exam (we also tell them that their teachers won't see their writing and that no one will be able to link it to them):[24]

> *Take the next several minutes to write as openly as possible about your thoughts and feelings regarding the exam you are about to take. In your writing, really let yourself go and explore your emotions and thoughts as you are getting ready to start the exam. You might relate your current thoughts to the way you have felt during other similar situations at school or in other situations in your life. Please try to be as open as possible as you write about your thoughts at this time.*

Think carefully about what to say when students struggle. When a student struggles with math (or any subject), it's natural to want to console him. You can see he's frustrated and unhappy, and you want to help him feel better. But consoling the student—by saying, for example, "It's OK, not everyone can be good at these types of problems"—may send the wrong message. The student may understand the subtext to be, "You've failed, and I am really sorry about that, but I'm not contradicting your conclusion that this math work is too hard for you." Consolation sends a subtle message that validates the student's opinion that he's not good at math, and can lower a student's motivations and expectations for future performances.

A better message is only slightly different: "Yes, this work is challenging, but I know that with hard work you can do it!" This acknowledges the student's experience—there's no sugarcoating the fact that he can't do it—but it expresses confidence that he has the capability. Also, giving concrete strategies for changing up study habits or for approaching a particular problem differently in the future helps him understand that, with added hard work and effort, he has the potential for success.[25] □

When students struggle, teachers should acknowledge that the work is challenging but that they can do it.

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