## AN

eclucator
A-QUARTERTY UOURNAL OFEDUUCATIONAE RESEARCHEAND IDEAS

## The Sopfisticated Side of Elementary Mathematics



Solving the Achievement Puzzle

24
Art to Renew America's Soul

Growing with Lesson Study

## 35

Learning Science:
Content-With Reason

## Where Does <br> Freedom Ring?

Find out on Democracy Web, which offers teachers background information on the elements of democracy and uses comparative studies to highlight what freedom is-and what it is not.


Welcome to Democracy Webt







To prepare lessons on freedom and democracy, draw on Democracy Web's study guide for teachers and order your free poster of the Map of Freedom (which is updated annually with the global Freedom in the World survey). Then, go deeper by guiding your students in using the interactive Map of Freedom at wwww.democracyweb.com.

To request a free 2009 Map of Freedom, contact Elizabeth Floyd at 212/514-8040, ext. 12, or floyd@freedomhouse.org.


## 4 What's Sophisticated about Elementary Mathematics? Plenty—That's Why Elementary Schools Need Math Teachers By Hung-Hsi Wu

Improving mathematics instruction is a priority in the United States, but there's little agreement on how to do it. Here's an idea that is rarely discussed: starting no later than fourth grade, math should be taught by math teachers (who teach only math). Teaching elementary math in a way that prepares students for algebra is more challenging than many people realize. Given the deep content knowledge that teaching math requires-not to mention the expertise that teaching reading demands-it's time to reconsider the generalist elementary teacher's role.

## 9 Understanding Place Value

10 Teaching the Standard Algorithms

## 12 Defining Fractions



## 2 Mailbox

## 15 Piece by Piece

How Schools Solved the Achievement Puzzle and Soared By Karin Chenoweth

After five years of visiting highpoverty and high-minority schools that have demonstrated success, Karin Chenoweth has noticed a handful of key characteristics that these schools share: genuine teacher collaboration, a sharp focus on what students must learn, assessments that inform instruction, and strong relationships between adults and children.

## 20 A Model Solution

## 24 Coaxing the Soul of America Back to Life

## How the New Deal Sustained, and Was Sustained by, Artists By Roger G. Kennedy

During the Great Depression, thousands of artists were hired to depict "the American Scene." While the works revealed much suffering, they also captured the hard-working, self-reliant spirit of the people.

## 30 Growing Together

American Teachers Embrace the Japanese Art of Lesson Study By Jennifer Dubin

Lesson study is a form of professional development in which teachers work together to develop a
lesson and think about how students learn. The point is not the resulting lesson so much as what teachers learn as they study the content, consider instructional methods, and reflect on how their chosen approaches influence student understanding.

## 35 Learning Science <br> Content-With Reason

By Paul R. Gross
A recent study claimed that learning scientific content does not give students an edge in scientific reasoning. But the preponderance of the evidence clearly indicates that learning scientific content does enhance scientific reasoning-and students and scientists need both.


RANDI WEINGARTEN
President
ANTONIA CORTESE
Secretary-Treasurer
LORRETTA JOHNSON
Executive Vice President
© AMERICAN FEDERATION OF TEACHERS, AFL-CIO 2009

Cover Illustration by Roland Sarkany

## LISA HANSEL Editor <br> JENNIFER DUBIN Assistant Editor <br> JENNIFER BERNEY <br> Production/Editorial Assistant <br> SEAN LISHANSKY Editorial Assistant GLENNA SABO Editorial Intern JENNIFER CHANG Graphic Designer Jane feller Copyeditor

AMERICAN EDUCATOR (USPS 008 462) is published quarterly by the American Federation of Teachers 555 New Jersey Ave. N.W. Washington, DC 20001-2079 Phone: 202/879-4400 www.aft.org
Letters to the editor may be sent to the address above or to amered@aft.org.
AMERICAN EDUCATOR cannot assume responsibility for unsolicited manuscripts.
Please allow a minimum of four weeks for copyright permission requests.

Signed articles do not necessarily represent the viewpoints or policies of the AFT.

Although advertisements are screened as carefully as possible, acceptance of an advertisement does not imply AFT endorsement of the product or service.

AMERICAN EDUCATOR is mailed to all AFT teacher and higher education members as a benefit of membership. Non-AFT members may subscribe by mailing $\$ 10$ per year by check or money order to the address on the left.

MEMBERS: To change your address or subscription, notify your local union treasurer.

POSTMASTER: Send address changes to AMERICAN EDUCATOR
555 New Jersey Ave. N.W. Washington, DC 20001-2079

Periodicals postage paid at Washington, D.C., and additiona mailing offices.
ADVERTISERS: Contact
Karen Dorne Media Sales

## Readers Say Yes to Community Schools

Richard Rothstein's article, "Equalizing Opportunity: Dramatic Differences in Children's Home Life and Health Mean That Schools Can't Do It Alone," was one of the best articles on the topic I have ever read. I have been teaching in the Philadelphia Public Schools since 1981, and I have seen or experienced the frustrations associated with everything Mr. Rothstein mentions.


I read with great interest Richard Rothstein's article in the Summer issue. He rightly points out that students can't learn if they are not consistently healthy or well fed. What was not mentioned in the article is that students can't learn if their emotional needs are not met in a consistent manner. Unsettled home situations hamper many a smart student's ability to succeed in school, and that is what I see more often than not.
-RACHEL HORWITZ
McKinley Middle School Albuquerque, N.M.

Thank you for devoting the Summer issue to community schools across the country, especially at this time when schools are under increasing pressure to meet the increasing needs of students, families, and communities, and to do so with smaller budgets and fewer resources. Schools cannot do it aloneschool/community partnerships are necessary.

In Illinois, community school work is having positive impacts on students and families throughout the state. The Federation for Community Schools has identified more than 250 community schools in the state, including roughly 150 in Chicago, as well as nearly 100 more school/community partnerships ready to undertake the transformation to community schools.

Recently, the Illinois General Assembly passed a community schools bill, HB 684, that amends the state's school code to include community schools. The bill's success isn't just a mark of the great community school work going on in

Illinois, but is also a sign of the support that community schools have from legislators and policymakers alike.

We also believe that by removing barriers to student success outside the classroom, community schools enable children and young people to be better learners in the classroom, and also enable teachers to focus on what they love and do bestteach. We were thrilled to see the AFT spread this message with its members, too.
-SUZANNE ARMATO
The Federation for Community Schools Chicago, III.

## All Right Already

As an English teacher, I was disappointed when I turned to page 8 of the Summer 2009 issue. I was surprised to see "all right" spelled as one word, and in the title, no less! According to the American references with which I am familiar, this is still not accepted as standard. Please accept my stickler's plea for correct usage, as my heart may not be able to take another shock. Other than that, I find your publication refreshing, inspirational, and current.
-SHELLY NEAL
Blackford County Schools Hartford City, Ind.

[^0]for standard spellings.

## Spring Issue Provides Insights

 To use New York City's numerical report card scores for American Educator articles is to consistently mark in 3.5 (more than meeting the professional expectation) to 4 (exceeding the standard). The Spring 2009 edition is a straight 4 cover to cover. Daniel T. Willingham is always a genius for making us think-no matter how hard it is-and the contribution by Rothstein, Jacobsen, and Wilder, "Grading Education," takes a teaching moment spotlight for a quote that should be on the front door of every school building. "Teachers are expected to repeat the mantra 'all children can learn,' a truth carrying the false implication that the level to which children learn has nothing to do with the out-of-school supports they receive." My professional mantra, which leads my lesson plans, is: it is not a question of if and can children learn, it is a matter of what and when they learn and how will they apply their knowledge to our purposes here.-DALE BENJAMIN DRAKEFORD
P.S. 132 and P.S. 204 Bronx, N.Y.

I thought I might be able to add something to "Dispelling Myths about Teacher 'Tenure,' " which appeared in the Spring 2009 issue. I have been in this profession a long time, having been a teacher, a custodian, and a superintendent. I know the value of having a union, especially in public education.

In 1960, I began my first classroom job. Many of us were recent military veterans and the need to have better income was a major concern. But boards of education were resistant to any pleas for help. Teacher presence at board meetings was often met with veiled threats of contract nonrenewal. The need for a strong united voice, along with a fair and consistent evaluation policy, was actually created by the unfair labor practices and lack of respect for the teaching profession by both elected board members and school administrators that existed in those days. We were dispensable. While low pay was an issue, the "take it or leave it policy" stayed in effect until the unions and collective bargaining were established.

After retiring as a school superinten-
dent, I returned to the classroom. I joined our local association and the national association because I know a united voice is needed. I also know that these associations have established fair evaluation procedures and fair labor practices. Neither of these protects the ineffective or poor-performing teacher if the administration is doing its job.
> -JIM RUBRIGHT
> Three Oaks Middle School Fort Myers, Fla.

I found the article, "Why Don't Students Like School?" by Daniel T. Willingham (Spring 2009) quite interesting because it confirms some of the things I have been observing for a long time. As an experienced secondary mathematics educator, I have concluded that the big movement in secondary math education toward emphasizing "critical thinking" will fail if students are not well grounded with mathematical background knowledge. So unless this background knowledge is learned first, students cannot possibly solve any problem successfully no matter how much they think about it.

The prerequisite knowledge may consist of knowing properties of numbers such as the distributive property (used in an example in the article) or the procedure for how to factor a specific polynomial. As the author points out, thinking occurs when a student combines information from the "environment" and her long-term memory in new ways.
-EDWARD ESPARZA
University of Texas at San Antonio San Antonio, Texas

I want to commend you for including vocational education as one of the goals for public education in your article "Grading Education," which appeared in the Spring 2009 issue. The public high schools that only offer college-prep courses are denying meaningful education for the students who do not plan on going to college. Instead of offering a college-prep program watered down to accommodate all students, we should offer equally rigorous options to academic and nonacademic students.
-JULIA FONG
Lincoln High School San Francisco, Calif.

Receive a 30\% Tuition Reduction on accredited graduate education programs

IOWLEDGE

## AFT Members Benefit at Drexel Online

The American Federation of Teachers partners with Drexel University Online to bring you a special AFT Members Plus Benefit! You and your immediate family members are entitled to receive a $30 \%$ tuition reduction on all graduate education programs and a 10-25\% tuition reduction on all other programs at Drexel Online.
kathryn.pischke@drexel.edu | (215) 895-0513

## Learn More:

drexel.com/aft-benefits
There's no cost to apply online!

A Union of Professionals
$\mathrm{AFT}+$
Member Benefits

# Whats50plisticatec about Elementaly Valmematics? 

## Plenty That's Why Elementary Schools Need Math Teachers



By Hung-Hsi Wu

Some 13 years ago, when the idea of creating a cadre of mathematics teachers for the upper elementary grades (who, like their counterparts in higher grades, would teach only mathematics) first made its way to the halls of the California legislature, the idea was, well, pooh-poohed. One legislator said something like: "All you have to do is add, subtract, multiply, and divide. How hard is that?"

The fact is, there's a lot more to teaching math than teaching how to do calculations. And getting children to understand impor-

Hung-Hsi Wu is a professor emeritus of mathematics at the University of California, Berkeley. He served on the National Mathematics Advisory Panel and has written extensively on mathematics textbooks and teacher preparation. Since 2000, he has conducted professional development institutes for elementary and middle school teachers. He has worked with the state of California to rewrite its mathematics standards and assessments, and was a member of the Mathematics Steering Committee that contributed to revising the math framework for the National Assessment of Educational Progress. He was also a member of the National Research Council's Mathematics Learning Study Committee.
tant ideas like place value and fractions is hard indeed.
As a mathematician who has spent the past 16 years trying to improve math education-including delivering intensive professional development sessions to elementary-grades teachers-I am an advocate for having math instruction delivered by math teachers as early as possible, starting no later than fourth grade.* But I also understand that until you appreciate the importance and complexity of elementary mathematics, it will not be apparent why such math teachers are necessary.

In this article, I address two "simple" topics to give you an idea of the advanced content knowledge that is needed to teach math effectively. Our first topic-adding two whole numbers-is especially easy. The difficulty here is mostly in motivating and engaging students so that they come to understand the standard addition algorithm and, as a result, develop a deeper appreciation of place value (which is an absolutely critical topic in elementary

[^1]math). This discussion of addition may not convince you that math teachers are a necessity in the first through third grades, but it will give you a deeper appreciation of the important mathematical foundation that is being laid in the early grades.

Our second topic-division of fractions-is substantially harder, though it's still part of the elementary mathematics content as it should be taught in fifth and sixth grades. This is a topic that, in my experience, many adults struggle with. My goal here is twofold: (1) to show you that elementary math can be quite sophisticated, and (2) to deepen your knowledge of division and fractions. Along the way, I think it will become apparent why mathematicians consider facility with fractions essential to, and excellent preparation for, algebra. By the end, I hope you will join me in calling for the creation of a cadre of teachers who specialize in the teaching of mathematics in grades 4-6. For simplicity, we will refer to them as math teachers, to distinguish them from elementary teachers who are asked to teach all subjects.

## Adding Whole Numbers

Consider the seemingly mundane skill of adding two whole numbers. Take, for example, the following.

45
$+31$
76
Nothing could be simpler. This is usually a second-grade lesson, with practice continuing in the third grade. But if you were the teacher, how would you convince your students that this is worth learning? Too often, children are given the impression that they must learn certain mathematical skills because the teacher tells them they must. So they go through the motions with little personal involvement. This easily leads to learning by rote. How, then, can we avoid this pitfall for the case at hand? One way is to teach them what it means to add numbers, why it is worth knowing, why it is hard if it is not done right, and finally, why it can be fun if they learn how to add the right way.

All this can be accomplished if you begin your lesson with a story, like this: Alan has saved 45 pennies and Beth has saved 31. They want to buy a small package of stickers that costs 75 cents, and they must find out if they have enough money together. To act this out, you can show children two bags of pennies, one bag containing 45 and the other 31 . Now dump them on the mat and explain that they have to count how many there are in this pile. Chances are, they will mess up as they count. Let them mess up before telling them there is an easier way. Go back to the bags of 45 and 31, and explain to them that it is enough to begin with 45 and continue to count the pennies in the bag of 31 . In other words, to find out how many are in 45 and 31 together, start with 45 and just go 31 more steps; the number we land on is the answer. To show them that making these steps corresponds exactly to counting, do a simple case with them. If there are 3 pennies in the smaller bag instead of 31 , then going 3 steps from 45 lands at 48 because
$45 \rightarrow 46 \rightarrow 47 \rightarrow 48$.
So 48 is the total number of pennies in the two bags of 45 and 3. Now ask them to count like this for 45 and 31 ; chances are, most of them will find this a bit easier but many will still mess up. You can help them get to 76, but they probably will get frustrated. That is good: here is something they want to learn, but they find it is
not so easy.
Then you get to play the magician. Tell them that what they are doing is called "adding numbers." In this case, they are adding 31 to 45 , written as $45+31$ (teach them to write addition horizontally as well as vertically from the beginning), and what it means is that it is the number they get by starting with 45 and counting 31 more steps. Show them they do not have to count so strenuously to get the answer to $45+31$ because they can do two simple additions instead, one being $4+3$ and the other $5+1$, and these give the two digits of the correct answer 76.

> All whole-number computations are nothing but a sequence of single-digit computations artfully put together. This is the kind of thinking students will need to succeed in algebra and advanced mathematics.

You can demonstrate this effectively by collecting the 45 pennies and putting them into bags of 10 ; there will be 4 such bags with 5 stragglers. Do the same with the other 31 pennies. Then place these bags and stragglers on the mat again, and ask them how many pennies there are. It won't take long for them to figure out that there are $4+3$ bags of 10 , and $5+1$ stragglers.

They will figure out that 7 bags of 10 together with 6 stragglers total 76 again. Now ask them to compare counting the bags and stragglers with the magic you performed just a minute ago. If they don't see the connection (and some won't), patiently explain it to them. Of course, this is the time to review place value. (To better understand place value, and to prepare for the occasional advanced student, see the sidebar on page 9.) Then, you can use place value to explain that when they add the 4 bags of 10 to the 3 bags of 10 , they are actually adding 40 and 30 .

| 45 |
| ---: |
| +31 |
| 76 |$\rightarrow$| $40+5$ |
| ---: |
| $+30+1$ |
| $?$ |$\rightarrow \quad$| $40+5$ |
| ---: |
| $+30+1$ |
| $70+6$ |$\rightarrow \quad$| 45 |
| ---: |
| +31 |
| 76 |

Now, they will listen more carefully to your incantations of place value because you have given them more incentive to learn about this important topic.

As mentioned above, addition of whole numbers is done mainly in grades 2 and 3 . Often, the addition algorithm is taught by rote, but some teachers do manage to explain it in terms of place value, as we have just done. Many educators believe that the real difficulty of this algorithm arises when "carrying" is necessary, but conceptually, carrying is just a sidelight, a little wrinkle on the fabric. The key idea is contained in the case of adding without carrying. If we succeed in getting students to thoroughly understand addition without carrying, then they will be in an excellent position to handle carrying too. (However, in my experience, the standard textbooks and teaching in most second- or third-grade classrooms focus on carrying before students are ready, and that is a pity.)

Understanding the addition algorithm in terms of place valuefor example, that $45+31$ is $40+30$ and $5+1$-is appropriate for beginners, but it cannot stop there. The essence of the addition algorithm, like all standard algorithms, lies in the abstract understanding that the arithmetic computations with whole numbers, no matter how large, can all be reduced to computations with single-digit numbers. (For more on this, see the sidebar on page 10.) In other words, students' ultimate understanding of these algorithms must transcend place value to arrive at the recognition that all whole-number computations are nothing but a sequence of single-digit computations artfully put together. This is the kind of thinking students will need to succeed in algebra and advanced mathematics. More precisely, students should get to the point of recognizing that $45+31$ is no more than the combination of two single-digit computations, $4+3$ and $5+1$. Whether the 4 stands for 40 or 40,000 and the 3 stands for 30 or 30,000 is completely irrelevant.

To drive home this point, consider the following two addition problems.

$$
\begin{array}{rr}
45 & 45723 \\
+31 & +\mathbf{3 1 2 5 1} \\
\hline 76 & \mathbf{7 6 9 7 4}
\end{array}
$$

The problem on the left is the one we have been working with, and parts of the problem on the right are tantalizingly similar, except that the 4 and 5 in the first row are no longer 40 and 5 but 40,000 and 5,000, respectively. Similarly, the 3 and 1 in the second row are not 30 and 1 but 30,000 and 1,000, respectively. Yet, do the changes in the place values of these four single-digit numbers (4, 5,3 , and 1) change the addition? Not at all, because the result is still the same two digits, 7 and 6 , and that is the point.

We are now able to directly address the main concern of this article, which is the need for math teachers at least starting in grade 4 . In grade 4, the multiplication algorithm has to be explained. A teacher knowledgeable in mathematics would know that this is the time to cast a backward glance at the addition algorithm to make sure students finally grasp a real understanding of what this algorithm is all about: just a sequence of single-digit computations. Why is this knowledge so critical at this point? Because it leads seamlessly to the explanation of why students must memorize the multiplication table (of single-digit numbers) to automaticity before they do multidigit multiplication: in the same way that knowing how to add single-digit numbers enables them to add any two numbers, no matter how large, knowing how to multiply single-digit numbers enables them to multiply any two numbers, no matter how large. We want students to be exposed, as early as possible, to the idea that beyond the nuts and bolts of mathematics, there are unifying undercurrents that connect disparate pieces.

Let us go a step further to make explicit the role of single-digit computations in the additions of $45+31$ and $45723+31251$. If students have been given the proper foundation in second grade, then in fourth grade, a math teacher will be able to give the following explanation.

$$
\begin{aligned}
45+31 & =(4 \times 10)+5+(3 \times 10)+1 \\
& =(4 \times 10)+(3 \times 10)+5+1 \\
& =(4+3) \times \mathbf{1 0}+(5+1)
\end{aligned}
$$

In the last equality, we used the distributive law-i.e., $(\mathrm{b}+\mathrm{c}) \mathrm{a}=$ ba + ca-to rewrite $(4 \times 10)+(3 \times 10)$ as $(4+3) \times 10$. For $45723+$

31251, we will focus only on 45 and 31 to enhance clarity. We have, then, the following.

$$
\begin{aligned}
45723+31251= & (4 \times 10000)+(5 \times 1000)+\cdots \\
& +(3 \times 10000)+(1 \times 1000)+\cdots \\
= & (4 \times 10000)+(3 \times 10000)+\cdots \\
& +(5 \times 1000)+(1 \times 1000)+\cdots \\
= & (4+3) \times \mathbf{1 0 0 0 0}+(5+1) \times \mathbf{1 0 0 0}+\cdots
\end{aligned}
$$

Again, the last equality makes use of the distributive law. If we compare the two expressions $(4+3) \times 10+(5+1)$ and $(4+3) \times$ $10000+(5+1) \times 1000$, we see clearly that the same single-digit additions $(4+3)$ and $(5+1)$ are in both of them, and that the difference between these expressions lies merely in whether these single-digit sums are multiplied by 10 or 1,000 or 10,000 , the place values of the respective digits. This clearly illustrates the primacy of single-digit computations in the addition algorithm.

Returning to our original second-grade lesson of $45+31$, let's review what you have accomplished. You have shown students what addition means; this is important because we want to promote the good practice among students that through precise definitions, they get to know what they will do before doing it. Then you made them want to learn it, and made them realize that the most obvious method (counting) is not the easiest. Best of all, you
 opened their eyes to the magic of learning: acquiring the power of making something complicated much simpler. Instead of tedious, error-prone counting, you used the concept of place value to introduce the idea of breaking up a task digit by digit and adding only two single-digit numbers in succession. A couple of years later, the fourth-grade math teacher will have the opportunity to explain and make explicit the idea that to add any whole numbers, no matter how large, all the children need to do is add single-digit numbers.

The main goal of the elementary mathematics curriculum is to provide children with a good foundation for mathematics. In this context, the addition algorithm, when taught as described above in grades 2-4, serves as a splendid introduction. It teaches children an important skill in mathematics: if possible, always break up a complicated task into a sequence of simple ones. This is why we do not look at 45 and 31 , but only 4 and 3 , and 5 and 1 .

Of course, they will encounter somewhere down the road something like $45+37$, but they will be in a position to understand that the carrying step is actually adding a 1 to the 10 s column. Despite how it is presented in most U.S. textbooks, carrying is not the main idea of the addition algorithm. The main idea is to break up any addition into the additions of single-digit numbers and then, drawing on our understanding of place value, put these simple computations together to get the final answer. If you can make your students understand that, you are doing fantastically well as a teacher, because you have taught them important mathematics. They now have an important skill and know
the reasoning behind it-and they will have used both to deepen their appreciation of place value.

## Dividing Fractions

I've had plenty of encounters with well-educated adults who can't divide fractions without a calculator, or who can, but have no idea why the old rule "invert and multiply" works. With that in mind, I'll break this topic into three parts: we'll review division, then fractions, and finally the division of fractions. Along the way, the answer to our larger question-what's sophisticated about elementary mathematics?-will become apparent, as will the ways in which mastering fractions prepares students for algebra.

Let's begin with the division of whole numbers, which would

> We want students to be exposed, as early as possible, to the idea that beyond the nuts and bolts of mathematics, there are unifying undercurrents that connect disparate pieces.

normally be taught in third grade. What does $24 / 6=4$ mean? In the primary grades, we teach two meanings of division of whole numbers: partitive division* and mea-
surement division. For brevity, let us concentrate only on measurement division, in which the meaning of $24 / 6=4$ is that by separating 24 into equal groups of 6 , we find that there are 4 groups in all. So the quotient 4 tells how many groups of 6 s there are in 24.

By fifth grade, students should be ready to apply their understanding of measurement division to a more symbolic format. This will prepare them for the division of fractions, for which the idea of "dividing into equal groups" often is not very helpful in calculating answers. (For example, the division of $1 / 7$ by $1 / 2$ does not lend itself to any easy interpretation of dividing $1 / 7$ into equal groups of $1 / 2$. Being able to draw or visualize where $1 / 7$ and $1 / 2$ fall on the number line is helpful in estimating the answer, but not in arriving at the precise answer, $2 / 7$.) Any understanding of fraction division, therefore, has to start from a more abstract level. With this in mind, we express the separation of 24 objects into 4 groups of 6 s symbolically as $24=6+6+6+6$, which is, of course, equal to $4 \times 6$, by the very definition of whole-number multiplication. Thus, the division statement $24 / 6=4$ implies the multiplication statement $24=4 \times 6$.

At this point, we must investigate whether the multiplication statement $24=4 \times 6$ captures all of the information in the division statement $24 / 6=4$. It does, because if we know $24=4 \times 6$, then we know $24=6+6+6+6$, and therefore 24 can be separated into 4 groups of 6 s . By the measurement meaning of division, this says $24 / 6=4$. Consequently, the multiplication statement $24=4 \times 6$ carries exactly the same information as the division statement $24 / 6=4$. Put another way, the meaning of $24 / 6=\mathbf{4}$ is $24=\mathbf{4} \times 6$. This is the symbolic reformulation of the concept of division of whole numbers that we seek.

This meaning of division is actually very clear from the stan-

[^2]dard algorithm for long division, as shown in the following example.
$6 \longdiv { 2 4 }$
-24
0
What we tell children is that to divide 24 by 6 , we look for the number which, when multiplied by 6 , gives 24 . (Of course, children who have memorized the multiplication table of single-digit numbers will do this easily; those who haven't will struggle.)

In a similar fashion, the meaning of $36 / 12=\mathbf{3}$ is that $36=\mathbf{3} \times 12$, and the meaning of $252 / 9=\mathbf{2 8}$ is that $252=\mathbf{2 8} \times 9$, etc. There is a subtle point here that is usually slurred over in the upper elementary grades but should be pointed out: in our examples, the dividend (be it 24,36 , or 252 ) is a multiple of the divisor, since otherwise the quotient cannot be a whole number. That said, now we can use abstract symbols ${ }^{\dagger}$ to express this new understanding of the division of whole numbers as follows: for whole numbers $m$ and $n$, where $m$ is a multiple of $n$ and $n$ is nonzero, the meaning of the division $\mathrm{m} / \mathrm{n}=\mathbf{q}$ is that $\mathrm{m}=\mathbf{q} \times \mathrm{n}$.

Beginning in fifth grade, we should teach students to reconceptualize division from this point of view. Their math teachers should help them revisit division from the perspective of this new knowledge and reshape their thinking accordingly. Such is the normal progression of learning.

Note that this reconceptualization is not a rejection of students' understanding of the division of whole numbers in their earlier grades. On the contrary, it evolves from that understanding and makes it more precise. This reconceptualization is important because the meaning of division, when reformulated this way, turns out to be universal in mathematics, in the following sense: if $m$ and $n$ are any two numbers (i.e., not just whole numbers) and $n$ is nonzero, then the definition of " $m$ divided by $n$ equals $q$ " is that $\mathrm{m}=\mathrm{q} \times \mathrm{n}$. In other words, $\mathrm{m} / \mathrm{n}=\mathbf{q}$ means $\mathrm{m}=\mathbf{q} \times \mathrm{n}$.

We now turn to fractions, a main source of math phobia. In the early grades, grades 2-4 more or less, students mainly acquire the vocabulary of fractions and use it for descriptive purposes (e.g., $1 / 4$ of a pie). It is only in grades 5 and up that serious learning of the mathematics of fractions takes place-and that's when students' fear of fractions sets in.

From a curricular perspective, this fear can be traced to at least two sources. The first is the loss of a natural reference point when students work with fractions. In learning to deal with the mathematics of whole numbers in grades $1-4$, children always have a natural reference point: their fingers. But for fractions, the curricular decision in the United States has been to use a pizza or a pie as the reference point. Unfortunately, while pies may be useful in the lower grades, they are an awkward model for fractions bigger than 1 or for any arithmetic operations with fractions. For example, how do you multiply two pieces of pie or use a pie to solve speed or ratio problems?

A second source of the fear of fractions is the inherently abstract

[^3]nature of the concept of a fraction. Whereas students' intuition of whole numbers can be grounded in counting their fingers, learning fractions requires a mental substitute for their fingers. By its very nature, this mental substitute has to be abstract because most fractions (e.g., ${ }^{19} / 13$ or ${ }^{251} / 604$ ) tend not to show up in the real world.

Because fractions are students' first serious excursion into abstraction,* understanding fractions is the most critical step in understanding rational numbers ${ }^{\dagger}$ and in preparing for algebra. In order to learn fractions, students need to know what a fraction is. Typically, our present math education lets them down at this critical juncture. All too often, instead of providing guidance for students' first steps in the realm of abstraction, we try in every conceivable way to ignore this need and pretend that there is no

> Because fractions are students' first serious excursion into abstraction, understanding fractions is the most critical step in preparing for algebra.

abstraction. When asked, what is a fraction?, we say it is just something concrete, like a slice of pizza. And when this doesn't work, we continue to skirt the question by offering more metaphors and more analogies: What about a fraction as "part of a whole"? As another way to write division problems? As an "expression" of the form $\mathrm{m} / \mathrm{n}$ for whole numbers m and $\mathrm{n}(\mathrm{n}>0)$ ? As another way to write ratios? These analogies and metaphors simply don't cut it. Fractions have to be numbers because we will add, subtract, multiply, and divide them.

What does work well for showing students what fractions really are? The number line. In the same way that fingers serve as a natural reference point for whole numbers, the number line serves as a natural reference point for fractions. ${ }^{\ddagger}$ The use of the number line has the immediate advantage of conferring coherence on the study of numbers in school mathematics: a number is now defined unambiguously to be a point on the number line. ${ }^{\S}$ In particular, regardless of whether a number is a whole number, a fraction, a rational number, or an irrational number, it takes up its natural place on this line. (For the definition of fractions, including how to find them on the number line, see the sidebar on page 12.)

Now, let's describe the collection of numbers called fractions. Divide a line segment from 0 to 1 into, let's say, 3 segments of equal length; do the same to all the segments between any two consecutive whole numbers. These division points together with the whole numbers then form a sequence of equal-spaced points. These are

[^4]the fractions with denominators equal to 3: the first division point to the right of 0 is what is called $\frac{1}{3}$, and the succeeding points of the sequence are then $2 / 3,3 / 3,4 / 3$, etc. The same is true for $1 / n, 2 / n, 3 / n$, etc., for any nonzero whole number $n$. Thus, whole numbers clearly fall within the collection of numbers called fractions. If we reflect the fractions to the left of 0 on the number line, the mirror image of the fraction $\mathrm{m} / \mathrm{n}$ is by definition the negative fraction - m/n. Therefore, positive and negative fractions are now just points on the number line. Most students would find marking off a point $1 / 2$ of a unit to the left of 0 to be much less confusing than contemplating a negative $1 / 2$ piece of pie.

The number line is especially helpful in teaching students about the theorem on equivalent fractions, the single most important fact in the subject. To state it formally, for all whole numbers $\mathrm{k}, \mathrm{m}$, and n (where $\mathrm{k} \neq 0$ and $\mathrm{n} \neq 0$ ), $\mathrm{m} / \mathrm{n}=\mathrm{km} / \mathrm{kn}$. In other words, $\mathrm{m} / \mathrm{n}$ and $\mathrm{km} / \mathrm{kn}$ represent the same point on the number line. Let us consider an example to get a better idea: suppose $m=4, n=3$, and $\mathrm{k}=5$. Then the theorem asserts that

$$
\frac{4}{3}=\frac{5 \times 4}{5 \times 3} \quad \text { and, of course, } \quad \frac{5 \times 4}{5 \times 3}=\frac{20}{15} .
$$

The number line makes the equality clear. To see how $4 / 3$ equals $20 / 15$, draw a number line and divide the space between 0 and 1 , as well as between 1 and 2, into three equal parts. Count up to the 4th point on the sequence of thirds-that's $4 / 3$. Then take each of the thirds and divide them into 5 equal parts (an easy way to make 15ths). Count up until you get to the 20 th point on the sequence of 15 ths-that's $20 / 15$, and it's in the same spot as $4 / 3$.


The use of the number line has another advantage. Having whole numbers displayed as part of fractions allows us to see more clearly that the arithmetic of fractions is entirely analogous to the arithmetic of whole numbers. For example, in terms of the number line, $4+6$ is just the total length of the concatenation (i.e., linking) of a segment of length 4 and a segment of length 6.


Then in the same way, we define $1 / 6+1 / 4$ to be the total length of the concatenation of a segment of length $1 / 6$ and a segment of length $1 / 4$ (not shown in proportion with respect to the preceding number line).


We arrive at $1 / 6+1 / 4=10 / 24$ as we would if we were adding whole numbers, as follows. Using the theorem on equivalent fractions, we can express $1 / 6$ and $1 / 4$ as fractions with the same denominator: $1 / 6=4 / 24$ and $1 / 4=6 / 24$. The segment of length $1 / 6$ is therefore the concatenation of 4 segments each of length $1 / 24$, and the segment of length $1 / 4$ is the concatenation of 6 segments each of length $1 / 24$. The preceding concatenated segment is therefore the concatenation of $(4+6)$ segments each of length $1 / 24$, i.e., ${ }^{10} / 24 .{ }^{* *}$ In this way,
(Continued on page 10)

[^5]
## Understanding Place Value

Many teachers, rightly in my opinion, believe place value is the foundation of elementary mathematics. It is often taught well, using manipulatives such as base-10 blocks to help children grasp that, for example, the 4 in 45 is actually 40 and the 3 in 345 is actually 300.

But despite the importance of place value, the rationale behind it usually is not taught in colleges of education or in math professional development. That's probably because the deeper explanation is not appropriate for most students in the first and second grades, which is when place value is emphasized. But it is appropriate for upper-elementary students who are exploring number systems that are not base 10 (which often is done, without enough explanation, through games)-and it is certainly something that math teachers should know. So here it is: the sophisticated side of the simple idea of place value.

Let's begin with a look at the basis of our so-called Hindu-Arabic numeral system. * The most basic function of a numeral system is the ability to count to any number, no matter how large. One way to achieve this goal is simply to make up symbols to stand for larger and larger numbers as we go along. Unfortunately, such a system requires memorizing too many symbols, and makes devising a simple method of computation impossible. The overriding feature of the HinduArabic numeral system, which will be our exclusive concern from now on, is the fact that it limits itself to using exactly ten symbols- $0,1,2,3,4,5,6,7$, 8,9 -to do all the counting. ${ }^{\dagger}$ Let us see, for example, how "counting nine times" is represented by 9 . Starting with 0 , we go nine steps and land at 9 , as shown below.

$$
0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 8 \rightarrow 9
$$

But, if we want to count one more time beyond the ninth (i.e., ten times), we would need another symbol. Since we are restricted to the use of only these ten symbols, someone long ago got the idea of placing these same ten symbols next to each other to create more symbols.

The most obvious way to continue the counting is, of course, to simply recycle the same ten symbols over and over again, placing them in successive rows, as follows.

0123456789
0123456789
0123456789

[^6]In this scheme, counting nine times lands us at the 9 of the first row, and counting one more time would land us at the 0 of the second row. If we want to continue counting, then the next step lands us at the 1 of the second row, and then the 2 of the second row, and so on.

However, this way of counting obviously suffers from the defect of ambiguity: there is no way to differentiate the first row from the second row so that, for example, going both two steps and twelve steps from the first 0 will land us at the symbol 2. The central breakthrough of the Hindu-Arabic numeral system is to distinguish these rows from each other by placing the first symbol (0) to the left of all the symbols in the first row, the second symbol (1)
to the left of all the symbols in the second row, the third symbol (2) to the left of all the symbols in the third row, etc.

| 00 | 01 | 02 | 03 | 04 | 05 | 06 | 07 | 08 | 09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 |
| 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| $\vdots$ | $\vdots$ |  | $\vdots$ |  | $\vdots$ |  | $\vdots$ |  |  |
| 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 |

Now, the tenth step of counting lands us at 10, the eleventh step at 11, etc. Likewise, the twentieth step lands us at 20, the twenty-sixth step at 26, the thirty-first step at 31, etc. By tradition, we omit the 0s to the left of each symbol in the first row. That done, we have re-created the usual ninety-nine counting numbers from 1 to 99.

We now see why the 2 to the left of the symbols on the third row stands for 20 and not 2 , because the 2 on the left signifies that these are numbers on the third row, and we get to them only after we have counted 20 steps from 0 . Similarly, we know 31 is on the fourth row because the 3 on the left carries this information; after counting thirty steps from 0 we land at 30, and one more step lands us at 31. So the 3 of 31 signifies 30 , and the 1 signifies one more step beyond 30 . With a trifle more effort, we can carry on the same discussion to three-digit numbers (or more). The moral of the story is that place value is the natural consequence of the way counting is done in the decimal numeral system.

For a fuller discussion, including numbers in arbitrary base, see pages 7-9 of The Mathematics K-12 Teachers Need to Know on my Web site at http://math. berkeley.edu/~wu/School mathematics1.pdf.
-H.W.
(Continued from page 8) students get to see that fractions are the natural extension of whole numbers and not some confusing new thing. This realization smoothes the transition from computing with whole numbers to computing with fractions.

Hopefully this discussion has smoothed the transition for you too, because it's time for us to skip ahead to sixth grade and tackle division with fractions. Having learned to add, subtract, and multiply with fractions, students should be comfortable with fractions as numbers (just like whole numbers). So, their learning to divide with fractions
can make use of the same scaffolding as learning to divide with whole numbers; students proceed from the simple to the complex. For example, a simple problem like $1 / 2 \div 1 / 4=2$ could be taught using the measurement definition of division and showing students on the number line that $1 / 4$ appears twice in $1 / 2$. That's fine as an introduction, but ultimately, in order to prepare for more advanced mathematics, students must grasp a more abstractand precise-definition of division with fractions. They must be able to answer the following question:

$$
\text { Why does } \frac{5}{6} \div \frac{9}{4} \text { equal } \frac{5}{6} \times \frac{4}{9} ?
$$

In other words, why invert and multiply? To give an explanation,

## Teaching the Standard Algorithms

In the context of school mathematics, an algorithm is a finite sequence of explicitly defined, step-by-step computational procedures that end in a clearly defined outcome. The so-called standard algorithms for the four arithmetic operations with whole numbers are perhaps the best known algorithms.

At the outset, we should make clear that there is no such thing as the unique standard algorithm for any of the four operations,,$+- x$, or $\div$, because minor variations have been incorporated into the algorithms by various countries and ethnic groups. Such variations notwithstanding, the algorithms provide shortcuts to what would otherwise be labor-intensive computations, while the underlying mathematical ideas always remain the same. Therefore, from a mathematical perspective, the label "standard algorithms" is justified.

While it is easy to see why these algorithms were of interest before calculators became widespread, a natural question now is why we should bother to teach them. There are at least two reasons. First, without a firm grasp of place value and of the logical underpinnings of the algorithms, it would be impossible to detect mistakes caused by pushing the wrong buttons on a calculator. A more important reason is that, in mathematics, learning is not complete until we know both the facts and their underlying reasons. For the case at hand, learning the explanations for these algorithms is a very compelling way to acquire many of the basic skills as well as the abstract reasoning that are integral to mathematics. Both these skills and the capacity for abstract reasoning are absolutely essential for understanding fractions, decimals, and, therefore, algebra in middle school. One can flatly state that if students do not feel comfortable with the mathematical reasoning used to justify the standard algorithms for whole numbers, then their chances of success in algebra are exceedingly small.

These algorithms also highlight one of the basic tools used by research mathematicians and scientists: namely, that whenever possible, one should break down a complicated task into simple subtasks. To be specific, the leitmotif of the standard algorithms is as follows: to perform a computation with multidigit numbers, break it down into several steps so that each step (when suitably interpreted) is a computation involving only single-digit numbers. Therefore, a virtue of the standard algorithms is that, when properly executed, they allow students to ignore the actual numbers being computed, no matter how large, and concentrate instead on single digits.

This is an excellent example of the kind of abstract thinking that is critical to success in mathematics learning.

Building on the discussion of the addition algorithm given in the main article, we can further illustrate this leitmotif with the multiplication algorithm. In this case, let us assume that students already know the meaning of multiplication as repeated addition. The next step toward understanding multiplication requires that they know the multiplication table by heart-i.e., that they know the multiplication of single-digit numbers to automaticity. We now show, precisely, how this knowledge allows them to compute the product $257 \times 48$. First, observe that $257=(2 \times 100)+(5 \times 10)+7$, so that by the distributive law [i.e., $a(b+c)=a b+a c]$ :

```
257\times4=(2\times4)\times100+(5\times4)\times10+(7\times4) and
257\times8=(2\times8)\times100+(5\times8)\times10+(7\times8).
```

Since they already know the single-digit products ( $2 \times 4$ ), $(5 \times 4),(7 \times 4),(2 \times 8),(5 \times 8)$, and $(7 \times 8)$, and they know how to add, they can compute $257 \times 4$ and $257 \times 8$. Such being the case, we further note that $48=(4 \times 10)+8$, so that again by the distributive law:

$$
257 \times 48=(257 \times 4) \times 10+(257 \times 8) .
$$

The right side being something they already know how to compute, they have therefore succeeded in computing $257 \times 48$ starting with a knowledge of the multiplication table. (For lack of space, we omit the actual writing out of the multiplication algorithm.)

Although the case of the long-division algorithm is more sophisticated, the basic principle is the same: it is just a sequence of single-digit computations.

For further details on the standard algorithms, see pages 38-90 of the first chapter of a professional development text for teachers that I am currently writing, available at http:// math.berkeley.edu/~wu/EMI1c.pdf.
-H.W.
we have to ask what it means to divide fractions in the first place. The fact that if we do not specify the meaning of dividing fractions, then we cannot possibly get a formula for it should be totally obvious, yet this fact is not common knowledge in mathematics education. For such a definition, let us go back to the concept of division for whole numbers. Recall that in the case of whole numbers, having a clearly understood meaning for multiplication (as repeated addition) and division (as measurement division) allowed us to conclude that the meaning of the division statement $\mathrm{m} / \mathrm{n}=\mathrm{q}$ for whole numbers $\mathrm{m}, \mathrm{n}$, and $\mathrm{q}(\mathrm{n}>0)$ is inherent in the multiplication statement $\mathrm{m}=\mathrm{q} \times \mathrm{n}$. But now we are dealing with fractions, and the situation is different. To keep this article from becoming too long, let's assume that we already know how to multiply fractions,* but we are still searching for the meaning of fraction division. Knowing that fractions and whole numbers are on the same footing as numbers, it would be a reasonable working hypothesis that if $\mathrm{m} / \mathrm{n}=\mathrm{q}$ means $\mathrm{m}=\mathrm{q} \times \mathrm{n}$ for whole numbers m , n , and q , then the direct counterpart of this assertion in fractions should continue to hold. Now, if $\mathrm{M}, \mathrm{N}$, and Q are fractions ( $\mathrm{N}>0$ ), we do not as yet know what $\mathrm{M} / \mathrm{N}=\mathrm{Q}$ means, although we know the meaning of $\mathrm{M}=\mathrm{Q} \times \mathrm{N}$ because we know how to multiply fractions. Therefore, the only way to make this "direct counterpart" in fractions come true is to use it as a definition of fraction division. In other words, we adopt the following definition: for fractions M and $\mathrm{N}(\mathrm{N}>0)$, the division of M by N , written $\mathrm{M} / \mathrm{N}$, is the fraction Q , so that $\mathrm{M}=\mathrm{Q} \times \mathrm{N}$.

We'll get acquainted with this definition by looking at a special case. Suppose

$$
\frac{5}{6} \div \frac{9}{4}=\mathrm{Q} \text { for a fraction } \mathrm{Q} \text {. }
$$

What could $Q$ be? By definition, this $Q$ must satisfy $5 / 6=Q \times 9 / 4$. Now, recalling that $\mathrm{m} / \mathrm{n}=\mathrm{km} / \mathrm{kn}$ (the theorem on equivalent fractions), we use this fact to find $Q$ by multiplying both sides of $5 / 6=$ Q $\times 9 / 4$ by $4 / 9$.

$$
\begin{aligned}
\frac{5}{6} \times\left(\frac{4}{9}\right) & =\mathrm{Q} \times \frac{9}{4} \times\left(\frac{4}{9}\right) \\
& =\mathrm{Q} \times\left(\frac{(9 \times 4}{4 \times 9}\right) \\
& =\mathrm{Q} \times 1=\mathrm{Q}
\end{aligned}
$$

This is the same as $5 / 6 \times 4 / 9=\mathrm{Q}$. We can easily check that, indeed, this $Q$ satisfies $5 / 6=Q \times 9 / 4$. So, we see that

$$
\frac{5}{6} \div \frac{9}{4}=\frac{5}{6} \times \frac{4}{9}
$$

and we have verified the invert-and-multiply rule in this special case. But the reasoning is perfectly general, and it verifies in exactly the same way that for a nonzero fraction $c / d$, if $(a / b) /(c / d)$ is equal to a fraction $Q$, then $Q$ is equal to $(a / b) \times(d / c)$. Therefore, the invert-and-multiply rule is always correct.

We have been staring at the concept of the division of fractions for quite a while, and we seem to be getting there because we have explained the invert-and-multiply rule. Therefore, it may be a little deflating to say that although we are getting very close, we are not quite there yet. There is a subtle point about the definition of fraction division that is still unsettled. This is something one

[^7]should probably not bring up in a sixth-grade classroom, but which is, nevertheless, something a math teacher should be aware of. The question is whether, for arbitrary fractions M and $\mathrm{N}(\mathrm{N}>0)$, we can always divide M by $\mathrm{N}-$ i.e., whether there is always a fraction Q so that $\mathrm{M}=\mathrm{Q} \times \mathrm{N}$. The answer, of course, is yes: if $M=a / b$ and $N=c / d$, then $Q=(a / b) \times(d / c)$ would do. So the upshot of all this is that we can always divide a fraction M by a nonzero fraction $N$, and the quotient, to be denoted by $\mathrm{M} / \mathrm{N}$, is the fraction obtained by the invert-and-multiply rule.

Once we know the meaning of division, we see there is nothing to the procedure of invert and multiply. What is sobering is that the rhyme, "Ours is not to reason why; just invert and multiply," gets it all wrong. With a precise, well-reasoned definition, there is

> The rhyme, "Ours is not to reason why; just invert and multiply," gets it all wrong. With a precise, well-reasoned definition, there is no need to wonder why-the answer is clear.

no need to wonder why-the answer is clear. Thus, we return to our earlier theme: before we do anything in mathematics, we must make clear what it is we are doing. In other words, we must have a precise definition of division before we can talk about its properties. (And we must have a precise definition of fractions before we can expect students to do anything with them.)

But one question remains: if division is just multiplication in a different format, why do we need division at all? The correct answer is that certain situations in life require it. An example of such a problem is the following:

A 5 -yard ribbon is cut into pieces that are each $3 / 4$ yard long to make bows. How many bows can be made?

Students usually recognize by rote that this problem calls for a division of 5 by $3 / 4$, but not the reason why division should be used. To better understand the reason for dividing, suppose the problem reads, instead, "A 30 -yard ribbon is cut into pieces that are each 5 yards long. How many pieces can be made?" It would follow from the measurement interpretation of the division of whole numbers that the answer is $30 / 5=6$ pieces-i.e., there are six 5 s in 30 . The use of division for this purpose is well understood.

However, we are now dealing with pieces whose common length is a fraction $3 / 4$, and the reason for solving the problem by dividing 5 by $3 / 4$ is more problematic for many students. But if we use the preceding definition of division, the reason emerges with clarity. Suppose Q bows can be made from the ribbon. Here Q could be a fraction, and the meaning of " $Q$ bows" can be explained by using an explicit example. If $Q=6 \frac{2}{3}$, for example, then " $6 \frac{2}{3}$ bows" means 6 pieces that are each $3 / 4$ yard long, plus a piece that is the length of 2 parts when the $3 / 4$ yard is divided into 3 parts of equal length. If multiplication is taught correctly, so that the multiplication of two fractions is defined clearly, one can then explain why $Q$ bows, no matter what fraction $Q$ is, have a total length of $Q$ $\times 3 / 4$ yards. Therefore, if $Q$ bows can be made from 5 yards of rib-
bon, then $5=\mathrm{Q} \times \frac{3}{4}$. By the definition of fraction division, this is exactly the statement that

$$
\frac{5}{3 / 4}=\mathrm{Q} .
$$

This is the reason why division should still be used to solve this problem. Incidentally, the invert-and-multiply rule immediately leads to $\mathrm{Q}=\frac{20}{3}$, which equals $6 \frac{2}{3}$ pieces. In greater detail, that's 6 pieces and a leftover piece that is the length of 2 parts when $3 / 4$ yard is divided into 3 equal parts.

## The Bigger Picture

At this point, I hope you can see that there's more to teaching elementary mathematics than is initially apparent. The fact is, there's much more to it than could possibly be covered in an article. But allow me to give you a glimpse of the bigger picture-of what elementary mathematics is really all about. I'll conclude with some of the latest thinking on the subject, thinking that points to mathematics teachers in the upper elementary grades being our best hope for providing all students the sound mathematics foundation they need.

Mathematics in elementary school is the foundation of all of K-12 mathematics and beyond. Therefore, to prepare students for
all that is to come, it must, in a grade-appropriate manner, respect the basic characteristics of mathematics. What does this mean? To answer this question, we have to remember that the school mathematics curriculum, beginning with approximately grade 5 , becomes increasingly engaged in abstraction and generality. It will no longer be about how to deal with a finite collection of numbers (such as, $1 / 2 \times(27-11)+56=$ ?), but rather about what to do with an infinite collection of numbers all at once (such as, is it true that $x^{4}+y^{4}=\left(x^{2}+y^{2}+\sqrt{2} x y\right)\left(x^{2}+y^{2}-\sqrt{2} x y\right)$ for all numbers $x$ and $y$ ?). The progression of the topics, from fractions to negative fractions, and on to algebra, Euclidean geometry, trigonometry, and precalculus, gives a good indication that to learn mathematics, a student gradually must learn to cope with abstract concepts and precise reasoning, and must acquire a coherent overview of topics that are, cognitively, increasingly complex and diverse. For this reason, students in the upper elementary grades must be prepared for the tasks ahead by being slowly acclimatized to coherence, precision, and reasoning, although always in a way that is grade appropriate. Allow me to amplify each of these characteristics below.

Coherence: If you dig beneath the surface, you will find that the

## Defining Fractions

The precise definition of a fraction as a point on the number line is a refinement of, not a radical deviation from, the usual concept of a fraction as a "part of a whole." As I will explain, this refinement produces increased simplicity, flexibility, and precision.

Let us begin with a line, which is usually taken to be a horizontal one, and fix two points on it. The one on the left will be denoted by 0 , and the one on the right by 1 . (Because we will not take up negative numbers, our discussion will focus entirely on the half-line to the right of 0 .) Now as we move from 0 to the right, we mark off successive points, each of which is as far apart from its neighbors as 1 is from 0 (like a ruler). Label these points by the whole numbers $0,1,2,3$, etc.


We begin with an informal discussion. If we adopt the usual approach to fractions, the "whole" would be taken to be the segment from 0 to 1 , called the unit segment, to be denoted by $[0,1]$. The number 1 is called the unit. Then a fraction such as $1 / 3$ would be, by common consent, 1 part when the whole $[0,1]$ is divided into 3 equal parts. So far so good. But if we try to press forward with mathematics, we immediately run into trouble because a fraction is a number-not a shape or a geometric figure. The unit segment $[0,1]$ therefore cannot be the whole. The language of "equal parts" is also problematic because in anything other than line segments, it usually is not clear what "equal parts" means. For example, if the whole is a ham, does "equal parts" mean parts with equal weights, equal lengths, equal amounts of meat, equal amounts of bones, etc.? So, we are forced to introduce more precision into our discussion in
order to avoid misunderstanding. What we should specify, instead, is that the whole is the length of the unit segment $[0,1]$, rather than the segment itself. When we say $[0,1]$ is divided into "equal parts," what we should say is that $[0,1]$ is divided into
 segments of equal length. The fraction $1 / 3$ therefore would be the length of any segment so that three segments of the same length, when pieced together, form a segment of length 1 . Since all segments between consecutive whole numbers have length 1 , when we likewise divide each of the segments between consecutive whole numbers into 3 segments of equal length, the length of each of these shorter segments is also $1 / 3$. In particular, each of the following thickened segments has length $1 / 3$ and is therefore a legitimate representation of $1 / 3$.


Now concentrate on the thickened segment on the far left. The distance of its right endpoint from 0 is naturally $1 / 3$. Since the value of each whole number on the number line reveals its distance from 0 (e.g., the distance of the point labeled 3 is exactly 3 from 0 ), logic demands that we label the right endpoint of this segment by the fraction $1 / 3$, and we call this
main topics of the elementary curriculum are not a collection of unrelated facts; rather, they form a whole tapestry where each item exists as part of a larger design. Unfortunately, elementary school students do not always get to see such coherence. For example, although whole numbers and fractions are intimately related so that their arithmetic operations are essentially the same, too often whole numbers and fractions are taught as if they were unrelated topics. The comment I frequently hear that "fractions are such different numbers" is a good indication that elementary mathematics education, as it stands, cannot go forward without significant reform, such as the introduction of math teachers. Another example of the current incoherence is the fact that finite decimals are a special class of fractions, yet even in the upper elementary grades, decimals often are taught as a topic separate from fractions. As a result, students end up quite confused having to learn three different kinds of numbers (whole numbers, fractions, and decimals), whereas learning about fractions should automatically make them see that the other two are just more of the same. These are only two of many possible examples of our splintered curriculum and the great harm it does to students' learning.

Another manifestation of the coherence of mathematics is the
ubiquity of the general principle of reducing a complicated task to a collection of simple subtasks. This principle runs right through all the standard algorithms, and also all the algorithms for decimals. In middle and high school mathematics, it also is the guiding principle in the discussion of congruence and similarity, provided these concepts are presented correctly. It also should be the guiding principle in the discussion of quadratic functions and their graphs, thereby making the basic technique of completing the square both enlightening and inevitable. Similarly, we saw how one embracing definition of division clarifies the meaning of the division of whole numbers and fractions and, as students should be taught in later grades, all rational numbers, real numbers, and complex numbers.

Precision: Children should learn about this mathematics tapestry in a language that does not leave room for misunderstanding or guesswork. It should be a language sufficiently precise so that they can reconstruct the tapestry step by step, if necessary. Too often, such precision of language is not achieved. For example, if you tell a sixth-grader that two objects are similar if they are the same shape but not necessarily the same size, it raises the question of what "same shape" means. A precise definition of similarity using
segment the "standard representation" of $1 / 3$. We also denote this thickened segment by $[0,1 / 3]$, because the notation clearly exhibits the left endpoint as 0 and the right endpoint as $1 / 3$. To summarize, we have described how the naive notion of $1 / 3$ as " 1 part when the whole is divided into 3 equal parts" can be refined in successive stages and made into a point on the number line, as shown below.


In a formal mathematical setting, we now use this particular point as the official representative of $1 / 3$. In other words, whatever mathematical statement we wish to make about the fraction $1 / 3$, it should be done in terms of this point. This agreement enforces uniformity of language and lends clarity to any mathematical discussion about $1 / 3$. At the same time, the preceding discussion also gives us confidence that we can relate this point on the number line to our everyday experience with $1 / 3$, should that need arise.

What we have done to the representation of $1 / 3$ can be done to any fraction with a denominator equal to 3 ; for example, the standard representation of $2 / 3$ would be the marked point to the right of $1 / 3$ on the line above, and that of $3 / 3$ would be 1 itself. In general, we identify any $\mathrm{m} / 3$ for any whole number $m$ with its standard representation, and we agree to let 0 be written as $\%$. Here, then, are the first several fractions with denominators equal to 3 .


Notice that it is easy to describe each of these fractions. For example, $7 / 3$ is the 7 th division point when the number line is divided into thirds (in self-explanatory language). Equivalently,
we can also say that $7 / 3$ is the 7 th multiple of $1 / 3$ (again, in self-explanatory language).

What we have done to fractions with denominators equal to 3 can be done to any fraction. In this way, we transform the naive concept of a fraction as a part of a whole into the clearly defined concept of a fraction as a point on the number line. There are many advantages of this indispensable transformation, but there are three that should be brought out right away.

On the number line, all points are on equal footing, so that in the preceding picture, for example, there is no conceptual difference between $2 / 3$ and $11 / 3$ because both numbers are equally easy to access. The essence of this message is that, when a fraction is clearly defined as a point on the number line, the conceptual difference between so-called proper and improper fractions completely disappears. So the first major advantage of understanding fractions as points on the number line is that all fractions are created equal. Now we can discuss all fractions all at once with ease, whether proper or improper. In this small way, the concept of a fraction begins to simplify, and learning about fractions gets easier.

The second major advantage is that such a concept of fractions is inherently flexible. Once we specify what the unit 1 stands for, all fractions can be interpreted in terms of the unit. Now we are ready for that ham. If we let 1 stand for the weight of the ham, then $\frac{1}{3}$ would represent a piece of ham that is a third of the whole ham in weight. If, on the other hand, we let 1 stand for the volume of the ham, then the same fraction will now be a piece of ham that is a third of the whole ham in volume-e.g., in cubic inches.

This brings us to the third major advantage: the increase in flexibility mandates an increase in precision. Gone is the loose reference to "equal parts" in such a setting, because one must ask, equal parts in terms of what unit?
-H.W.
the concept of dilation from a point $A$ would not allow for such confusion (as students will see that an object changes size, but not shape, when each point of the object is pushed away from or pulled into $A$ by the same scaling factor).

Another example of the need for precision manifests itself in the way we present concepts. It is worth repeating that before we do anything in mathematics, we must make clear what it is that we are doing by providing precise definitions. There is no better example of the need for precision than the way fractions are generally taught in schools. Too often, fractions are taught without definitions, so that students are always in the dark about what fractions are. Thus, students multiply fractions without knowing what multiplication means and, of course, they invert and multiply, but dare not ask why. It is safe to hypothesize that such conceptual opaqueness is largely responsible for the notorious

> It is unrealistic to expect our generalist elementary teachers to possess this kind of mathematical knowledge-especially considering the advanced knowledge they must acquire to teach reading.

nonlearning of fractions-and, as a result, for great difficulty as students begin algebra.

Reasoning: Above all, it is important that elementary school mathematics, like all mathematics, be built on reasoning. Reasoning is the power that enables us to move from one step to the next. When students are given this power, they gain confidence that mathematics is something they can do, because it is done according to some clearly stated, learnable, objective criteria. When students are emboldened to make moves on their own in mathematics, they become sequential thinkers, and sequential thinking drives problem solving. If one realizes that almost the whole of mathematics is problem solving, the centrality of reasoning in mathematics becomes all too apparent.

When reasoning is absent, mathematics becomes a black box, and fear and loathing set in. An example of this absence is some children's failure to shift successive rows one digit to the left when multiplying whole numbers, such as on the left below.

| 826 |
| ---: |
| $\times \quad 473$ |
| 2478 |
| 5782 |
| $+\quad 3304$ |
| 11564 |

$$
\begin{array}{r}
826 \\
\times \quad 473 \\
\hline 2478 \\
5782 \\
+\quad 3304 \\
\hline 390698
\end{array}
$$

If no reason is ever given for the shift, it is natural that children would take matters into their own hands by making up new rules. Worse, such children miss an excellent opportunity to deepen their understanding of place value and see that, in this example, the multiplication $4 \times 8$ is actually $400 \times 800$, and that this is the basic reason underlying the shift. Another notorious example is the addition of fractions by just adding the numerators and the denominators, something that happens not infrequently even in college.

Learning cannot take place in the classroom if students are kept in the dark about why they must do what they are told to do.

The characteristics of coherence, precision, and reasoning are not just niceties; they are a prerequisite to making school mathematics learnable. Too often, all three are absent from elementary curricula (at least as they are sketched out in both state standards and nationally marketed textbooks).* As a result, too often they also are absent from the elementary classroom. The fact that many elementary teachers lack the knowledge to teach mathematics with coherence, precision, and reasoning is a systemic problem with grave consequences. Let us note that this is not the fault of our elementary teachers. Indeed, it is altogether unrealistic to expect our generalist elementary teachers to possess this kind of mathematical knowledge-especially considering all the advanced knowledge of how to teach reading that such teachers must acquire. Compounding this problem, the pre-service professional development in mathematics is far from adequate. ${ }^{\dagger}$ There appears to be no hope of solving the problem of giving all children the mathematics education they need without breaking away from our traditional practice of having generalist elementary school teachers.

The need for elementary teachers to be mathematically proficient is emphasized in the recent report of the National Mathematics Advisory Panel. ${ }^{*}$ Given that there are over 2 million elementary teachers, the problem of raising the mathematical proficiency of all elementary teachers is so enormous as to be beyond comprehension. A viable alternative is to produce a much smaller corps of mathematics teachers with strong content knowledge who would be solely in charge of teaching mathematics at least beginning with grade 4 . The National Mathematics Advisory Panel has taken up this issue. While the absence of research evidence about the effectiveness of such mathematics teachers precluded any recommendation from that body, the use of mathematics teachers in elementary school was suggested for exactly the same practical reasons. ${ }^{\S}$ Indeed, this is an idea that each state should seriously consider because, for the time being, there seems to be no other way of providing our children with a proper foundation for mathematics learning.

We have neglected far too long the teaching of mathematics in elementary school. The notion that "all you have to do is add, subtract, multiply, and divide" is hopelessly outdated. We owe it to our children to adequately prepare them for the technological society they live in, and we have to start doing that in elementary school. We must teach them mathematics the right way, and the only way to achieve this goal is to create a corps of teachers who have the requisite knowledge to get it done.

[^8]
## Piece by Piece

## How Schools Solved the Achievement Puzzle and Soared



## By Karin Chenoweth

Most schools have traditionally been organized so that individual teachers operate in isolation, with no recognized standards for what or how to teach, and with only an occasional supervisor wandering through to criticize kids' behavior or teachers' bulletin boards. ${ }^{1}$ Good principals have taken great care in hiring teachers, but traditionally, a principal's job has been widely understood within

[^9]the education world to be handling and preventing crises, staving off parents by keeping them busy raising money for the school, and-at the high school level-producing winning sports teams. Superintendents are pretty much expected to do the same thing on a larger scale, which means they try to keep their school boards mostly focused on athletic fields and bond referenda instead of what and whether kids are learning.

That all sounds grim, but it gets worse. In general, teachers pretty much sink or swim-that is, become bad or good teacherson their own, with very little help from their colleges' teacher preparation programs, little help from principals and colleagues, and shockingly little guidance on what they are actually supposed to teach. ${ }^{2}$ "Teachers are born, not made," the old saw goes, implying that there is not really a body of knowledge and skill teachers need to master. Many a social studies teacher has been assigned to teach high school algebra with little more help than the airy sentiment, "A good teacher can teach anything."

As far as what they are supposed to teach, teachers have pretty
much had to make it up. They have rarely been provided a systematic plan of instruction that allows them to know what a student should have learned before getting to their classroom, what each student needs to learn in their classroom, and what the student will learn once he or she leaves their classroom. If they're lucky, they have colleagues who take pity on them and help out, but even then, the solutions are idiosyncratic, leaving far too many kids studying the rain forest and Charlotte's Web multiple times in their school careers without ever studying animal classification and Tom Sawyer.

By operating without clear standards for what they are supposed to teach or good information about how to ensure students learn, teachers-particularly inexperienced ones-are left to hope their kids arrive knowledgeable, disciplined, organized, and able to understand material the first time it is presented. Kids, being kids, rarely come in pre-educated, and children who grow up in poverty or isolation often arrive significantly behind in vocabulary, background knowledge, and organizational wherewithal. When kids arrive behind, they need much more skilled instruction than most middle-class kids require. The resulting disconnect between teacher hopes and reality leads to endless teacher frustration and is at least part of the reason so many young teachers flee high-poverty, high-minority schools in search of "better" kids or abandon the profession altogether. ${ }^{3}$

The sense that low student achievement in high-poverty and high-minority schools is the fault of the students themselves-and their families-has permeated the education profession. As a result, not only many teachers but also many principals, superintendents, academics, and even much of the public have come to think that there is little schools can do to help low-income students and students of color achieve at levels comparable to their more privileged peers. I disagree.

For the past five years, I have been visiting high-poverty and high-minority schools that have demonstrated success through their student achievement data.

Each school's reading, math, and science achievement data have been thoroughly examined to ensure that not only are the schools doing well in the aggregate, but that each group of students is also doing well. In these schools, achievement gaps are narrow or, in some cases, nonexistent. Aside from a few rudimentary checks to ensure that they have achieved their success legitimately, I simply ask the educators in those schools to describe what they do to achieve their success. My assumption is that they are the experts in their success, and that we need to learn what they have to teach. So it is all the more significant that I saw and heard about the same essential elements again and again.

Different principals and teachers list those elements in a different order and might use different words, but Molly BensingerLacy, principal of Graham Road Elementary School in Falls Church, Virginia,* was particularly succinct: "The strategies for educating students to high standards are pretty much the same for all kids: teacher collaboration; a laserlike focus on what we

[^10]want kids to learn; formative assessment to see if they learned it; data-driven instruction; personal relationship building."

In my new book, How It's Being Done, from which this article is drawn, I explore those essential elements and how I saw them play out in different schools and different contexts.

Anyone looking for simple answers will not find them here. As many of the teachers and administrators in these schools, which I call "It's Being Done" schools, have told me, there is no magic bullet-there is no single program, policy, or practice that will ensure all schools and all students will be successful. Educating children is a complex task, and when children live in poverty or isolation, the task is even more complex. If our nation is to have

> Children who grow up in poverty or isolation often arrive significantly behind in vocabulary, background knowledge, and organizational wherewithal.
an educated citizenry, we must be very thoughtful and deliberate about the way we structure all children's
 educational experiences. All the elements described below work together to fundamentally change how we go about educating all students.

## Teacher Collaboration

Many teachers, reading Bensinger-Lacy's recommendations for high standards of education, may say something along the lines of, "When are we supposed to collaborate? I teach all day, and during my planning times, I plan lessons and grade papers." Others may say, "We 'collaborate' [imagine air quotes and sarcastic tone], and it is such a waste of time. Then I have to go home and prepare lessons and grade papers until late at night." Both reactions are understandable in schools that do not provide the structures to make sure teacher collaboration is both possible and productive.

So let's begin at the beginning. The point of teacher collaboration is to improve instruction for students and to ensure that all students learn. No one teacher can be an expert in all aspects of the curriculum, all possible ways to teach it, and every child who sits in his or her class. But every teacher should have expertise that can be tapped by other teachers to improve their knowledge of their subject, their teaching skill, and their knowledge of their students.

It should be said, however, that learning from colleagues is not
something that is built into the field of American teaching. It sometimes springs up because teachers organize themselves to work together, but it has not been integral to teacher professional development or school organization. When teachers advise each other, consult with experts, think deeply about new ways to teach the material, and examine existing research in a systematic way in order to help all their students learn the material, they are working in sharp contrast to the way teachers have traditionally been expected to work. They are working in schools that have the structures and systems in place that make collaboration meaningful.

Let's examine the conditions necessary for the kind of collaboration I saw in It's Being Done schools.

## Time

I'm starting with the obvious, but that doesn't make it any less important. To make their time with students effective and worthwhile, teachers must have time to think about their lessons, observe each others' classes, examine student work, learn from colleagues and outside experts, and do all the other things that are subsumed under the term collaboration.

It's Being Done schools make sure that teachers have regular meeting times, usually during the course of the school day. The schools squeeze in the time where they can. Elementary schools generally schedule "specials"-that is, art, music, counseling, and physical education-so that all the students from a particular grade have them at one time, permitting the grade-level teachers time to meet. Some schools close early once a week to permit cross-grade collaborations. Others have aides start the school day, supervising the putting away of coats and boots, collecting homework and lunch money, and distributing backpack notices while teachers meet together. Many secondary schools schedule planning time so that the teachers can meet with their departments or teams. If possible, schools find money to pay teachers to stay after school or come in on Saturdays.

At Ware Elementary School in Fort Riley, Kansas, principal Deb Gustafson told me that when she speaks to other educators, the lack of available time to meet "is usually one of the biggest excuses." Since all schools have roughly the same amount of time, "The message needs to be that it has to be captured; creativity must be employed," she said.

The schools I visit are, for the most part, Title I schools, meaning that they receive federal funds aimed at high-poverty schools. As a result, they often have a bit more resources than non-Title I schools have to pay teachers to meet outside school hours or hire substitute teachers to allow for classroom observations. Not coincidentally, It's Being Done schools work hard to make sure that time with substitutes is not a waste of time for children. In Steubenville, Ohio, substitutes must get a minimum of one day of training in reading instruction and one day in math. In addition, each elementary school in the district is allocated 100 days of a substitute teacher; Wells Elementary hired a recently retired teacher for that part-time position.

One way or another, all of the schools carefully carve out time for teacher collaboration. But time is not enough. The time has to be well spent.

## Rules of Engagement

To make teacher collaboration time productive, cultural norms
about how that time will be spent must be established.

- If you don't say it in the meeting, don't say it in the parking lot. At Oakland Heights Elementary in Russellville, Arkansas, principal Sheri Shirley made this an explicit rule. Shirley wasn't looking to quell disagreements, but to ensure that they saw the light of day and didn't fester. Note, however, that this must be matched with openness on the part of the leader to hear things he or she might not want to hear.
- Focus discussions on the things the school can control rather than what it can't. Molly Bensinger-Lacy of Graham Road uses a graphic organizer for teachers to fill out all the causes
> "The strategies for educating students to high standards are: teacher collaboration; a laserlike focus on what we want kids to learn; formative assessment to see if they learned it; data-driven instruction; personal relationship building."

-MOLLY BENSINGER-LACY,
Principal of Graham Road Elementary School
of a given problem-and then together they cross out anything they don't have control over, from the poverty of the kids to the testing schedule of the district.

- Focus on specific objectives related to instruction. According to Ware Elementary's principal, Deb Gustafson, "meetings and requirements must be well organized, focused, agendadriven, and contain specific expectations." Meetings should not be filled with the administrative trivia of new roll-call systems, hall-duty assignments, or anything else that could be handled by e-mail.

At the beginning of the school improvement process, principals often will sit in on the teacher collaboration meetings to make sure the sessions are productive; once teachers have begun to internalize the norms, teachers usually meet on their own. Often principals will require that specific products result from these meetings, such as a curriculum map, formative assessment, or group of lesson plans complete with assignments.

And when teachers observe other classrooms, it is often with a specific aim in mind. In Elmont, New York, I learned about Elmont Memorial Junior-Senior High School's evaluation process, in which an "action plan" is formulated to help teachers improve. Here's one example: "By observing Ms. McDonnell, you will take note of smooth transitions between lesson activities that will enable you to maintain student attention. From Ms. Smith, you will see the perfect implementation and enforcement of sound opening strategies. Finally, from Mr. Schuler you will observe the benefits reaped from a well-structured activity." This is not simply sending teachers off to wander and possibly pick up some tips
from more experienced teachers, but rather a highly structured way of making sure teachers learn from each other.

## Good Teachers Willing to Collaborate to Improve Student Achievement

Again, so obvious you want to say, "Duh." But that doesn't make this an unimportant point. "You've got to have master teachers," said Susan Brooks, the principal who led the improvement of Lockhart Junior High School, in Lockhart, Texas. "It's all about teachers."

It's Being Done principals warn prospective teachers that they will be expected to work collaboratively. "Our interviews take a really long time," Bensinger-Lacy says, because she lays out in great detail the collaborative environment teachers will be

> While It's Being Done schools seek out accomplished teachers for tough assignments, they also recognize that someone just entering the profession needs a great deal of support.

expected to participate in. This has not made it difficult to recruit; on the contrary, as word gets around and success builds, most It's Being Done schools have found it easier to find applicants.

Although It's Being Done schools hire carefully-and occasionally counsel out teachers unwilling or unable to work collab-oratively-they also give good, experienced teachers time to get used to working in the kind of public way these schools require. One of the difficult issues involved in school improvement is that many veteran teachers are used to seeing a parade of one unsuccessful principal after another (not to mention superintendents), many of whom talk big before fizzling out. Those teachers need to be convinced that changing will be meaningful and not just another heartbreaking waste of time. That means there needs to be a commitment on the part of school leaders-who need the support of their superintendents-to stay in place for the improvement process. How long that takes depends on the school, but It's Being Done principals have told me that although there should be some signs of improvement, particularly in the school atmosphere, almost immediately, improvements in instruction might take as long as two or three years to be reflected in state test scores. To go from being the first school in Kansas to be put "on improvement" to one of the best schools in the state took Ware about six years; to go from being in the bottom third to the top third of schools in California took Imperial High School about as long.

Because the point of teacher collaboration is to improve student achievement, teachers in It's Being Done schools recognize that the students who struggle the most need the best teachers. At Wells Elementary, for example, one of the most accomplished reading teachers (in a building full of accomplished reading
teachers) is assigned to teach the "lowest" class of struggling firstgraders. This is in direct contrast to ordinary schools, where the best teachers are often rewarded with the "best" students, who are usually defined as those students who easily master new material with or without expert teachers.

While It's Being Done schools seek out accomplished teachers for tough assignments, they also recognize that someone just entering the profession, whether from a traditional or an alternative certification program, needs a great deal of support. "We got him as a baby, first rattle out," is the way Lockhart Junior High's Brooks described Jeffrey Knickerbocker, who came into teaching after working as a geophysicist. He himself said that when he first started, he was a "terrible teacher." Buthe got the help and support he needed and is now widely acknowledged both by his colleagues and by students to be among the best teachers in the school.

## Common Goals

Meaningful collaboration requires teachers to have meaningful things to collaborate about, and that is the subject of the next section. But even before that, teachers need to share the goal that every student be successful. Sometimes this means having the vision to see past their students' childhood and adolescent goofiness. English teacher José Maldonado at Granger High School in Granger, Washington, said this about his students, many of whom are tempted by the gangs that dominate the Yakima Valley: "I try to look beyond where they are now and see them for who they will be."

## A Laserlike Focus on <br> What We Want Kids to Learn

For generations, teaching has been an isolated activity, and teachers pretty much decided what they would teach. At the same time, teachers have long been whipsawed from one fad to another about how to teach. Teachers were told to keep their students seated in neat rows and columns, then they were told to have them sit in circles, and then in cooperative learning groups. They were told to have quiet classrooms, and then they were told to have lively yet controlled classrooms. And so on. Yet through all that, most teachers were still allowed to decide whether kids would learn about dinosaurs or the Bill of Rights. This is exactly backward. Teachers should be the experts in how to teach, but on their own, they should not be deciding what to teach.

After all, the reason we have schools is to impart the knowledge and skills that our society as a whole has deemed important. This means that decisions about what knowledge and skills children learn are of concern to all of us. That doesn't mean that there shouldn't always be room in a school day or year for teachers to share their passion for the more obscure plays of William Shakespeare. But the bulk of the curriculum should be devoted to the knowledge and skills that we as a society have decided are essential for students to become educated citizens.

Today, we are converging on the idea that every high school graduate should be ready for college or the workplace. The more we study what this actually means, the more we realize that the two are pretty much the same. To be ready for, say, a plumbing apprenticeship or to get a job on an automobile assembly line or as a sales representative requires that students have fairly high reading and writing levels and have mastered math at least
through Algebra II. In other words, students who are entering the workforce after high school require the same educational level as students who are ready for credit-bearing classes in college-at least if they want the kind of job that has traditionally offered paid vacation and health insurance.

The last 20 years has seen the beginning of agreement about what should be taught. For the most part, this has taken the form of states bringing together groups of teachers and content experts to set standards for what students are expected to know and be able to do by the time they graduate; then the groups work backward through the grades. The real problem is that too few states have done the hard job of developing clear, teachable standards. Some states have shied away from paring down what they want students to learn, so their standards tend to be impossibly large compendia of knowledge and skills. Other states have stuck with incredibly vague standards that do not offer any real guidance. Even in a field as seemingly definite as mathematics, the lack of clarity in standards has led to math curricula that are, as scholar William Schmidt says, "a mile wide and an inch deep."

By being too broad and expecting too much, many states essentially push the decisions of what to teach back onto individual teachers, who find themselves picking and choosing among standards rather than trying to teach all of them-because teaching all of them is impossible. (In contrast, by paring down the vast array of human knowledge into a relatively manageable yet ambitious set of standards, Massachusetts made a real contribution, and it did so long enough ago that those standards have really started permeating Massachusetts schools. Massachusetts now has the highest overall performance in reading and math on the National Assessment of Educational Progress.)

Many It's Being Done educators hope that all states and schools will eventually share the same ambitious national standards. As Ware's Gustafson told me in an e-mail: "National standards would help the students most in need, those with the highest mobility." She added that the difficulties of moving from school to school are compounded "by making the requirements different everywhere a student lands."

Even once common standards are embraced, however, teachers still have a lot of work to do. It's Being Done schools often have to build their own curriculum from scratch, and most spend quite a lot of time building "curriculum maps" or other documents that clearly delineate what each grade will study when. Roxbury Prep in Roxbury, Massachusetts, has teachers come in three weeks ahead of the students, in part to build that year's curriculum map. Graham Road Elementary School has daylong teacher retreats while students are taught by substitutes so that teachers can build their curriculum map, and Imperial High School has slowly built its curriculum map, subject by subject, over the years.

Once that initial planning is done, teachers don't have to start from scratch in subsequent years, but can work on improvements
and refinements each year. For this, they will often use the results on state tests. If their students didn't do well on measurement, for example, the teachers will revise their instructional strategies and may add time to that subject. If all the students have mastered standard punctuation, the teachers might decide to spend a little less time on that subject so they can add time to teaching students how to write research papers.

Teachers then work on how students should demonstrate their knowledge of the curriculum. To make this effective, teachers need to agree on a good assessment, what constitutes meeting standards, and what constitutes exceeding standards. Teachers often need help in learning how to do this work-which is known as proficiency setting or range finding-and in making sure that they are aiming at high standards (more on this topic in the next section, "Formative Assessments").

## It's Being Done schools often have

 to build their own curriculum from scratch, and most spend quite a lot of time building "curriculum maps" that clearly delineate what each grade will study when.Even now, teachers are not yet ready to walk into the classroom. A curriculum with assessments still isn't sufficient guidance for a teacher to know what he or she is doing tomorrow. Teachers in It's Being Done schools work together on lesson plans. This is where all their hard work in collaborating pays off for teachers. Because they work together so closely and because they are working on the same things at the same times, they are able to share the work of developing individual lessons. Outside the teaching profession, not everyone understands what a huge and complex burden lesson planning is-particularly for new teachers. At Lockhart Junior High School, new teachers are handed their entire first year of lessons so that they don't have to worry about planning. As Susan Brooks, the former principal, said, it takes so much effort to learn about the school's routines, culture, colleagues, and students-as well as to establish good classroom management and build relationships with their students-that new teachers simply don't have the time and energy to plan lessons. After their first year, they are welcomed into the collaborative process of lesson development. Far from feeling undermined, the new teachers I spoke to said they felt supported by this system.

## Formative Assessments

Students have always had regular assessments-I had weekly spelling and arithmetic tests all through my elementary school years, in addition to the big chapter tests, unit tests, and, of course,
(Continued on page 22)

## A Model Solution

For some schools, the smartest thing to do is adopt a school improvement model that has been demonstrated effective and then work hard to make it successful. No one should ever think this means those schools are not being creative. Symphony violinists do not compose their own music, but no one calls them uncreative. Ensuring that all children in a school are learningparticularly when the children live in poverty or isolationrequires creativity and thought at every juncture.

Today, we have quite a few successful, replicable models. In my 2007 book, It's Being Done, and in my new book, How It's Being Done (from which this sidebar is drawn), I profiled schools that have successfully used the Core Knowledge,
Success for All, and Uncommon Schools models. The Knowledge Is Power Program (KIPP) and Green Dot charter schools, which I have not visited but other authors have, appear to have developed still other successful school models.

But success is not guaranteed. Ware Elementary in Fort Riley, Texas, is an example of a school that used Success for All but was still unsuccessful until a real leader, Deb Gustafson, and her team arrived. So it is perfectly reasonable to want to save some trouble by adopting a carefully researched model, but making it work still requires energy, creativity, and knowledge.

In the brief excerpt below, we learn how P.S./M.S. 124, a K-8 school in Queens, New York, used the Core Knowledge model to move from an underperforming school to one in which seventh-graders sound like college students.

Did Shakespeare hate women?
The seventh-graders wondered. They had finished reading A Midsummer Night's Dream, and they couldn't agree. Heated arguments inspired the students to read more of Shakespeare's plays to try to answer the question. Some ended up answering yes, some no, depending on which plays they relied on, but the result was that the seventh grade of P.S./M.S. 124, otherwise known as Osmond A. Church School in Queens, New York, or just "P.S. 124," spent a lot longer on the Shakespeare unit than had been planned by their teachers. "It took on a life of its own," said principal Valarie Lewis.

To interest 12 -year-olds in formulating such a question, and then allow them to push their teachers for more time to read and use primary documents as evidence, is a worthy feat for any school. But P.S. 124 is a school that would be written off by some as incapable of nurturing such intellectual discourse because the vast majority of the students are minorities who qualify for free or reduced-price lunch. And yet, as a result of steady improvement over a number of years, the school posts higher proficiency rates than the state as a whole and much higher than New York

City. P.S. 124 began its improvement journey in 1999, when it received a three-year \$784,000 Comprehensive School Reform grant from the New York State Department of Education and the teachers and administrators agreed to adopt Core Knowledge, which was then a relatively new program.

Core Knowledge, conceived and developed by author and scholar E. D. Hirsch Jr., begins with the idea that it is the job of schools to produce educated citizens. To be educated means knowing a large body of content as preparation for being able to read, understand, and evaluate newspaper and magazine articles, election materials, jury instructions, scientific research, literature, and anything else educated citizens might be called upon to read and evaluate. The Core Knowledge Foundation has a plan for instruction that focuses on building a knowledge base about world history, geography, civics, literature, science, mathematics, art, and music. The federal grant paid for teachers to come in during the summer to learn the program. Core Knowledge gave a framework for teaching much more content than teachers had ever taught before. The teachers developed a three-month scope
 and sequence of what they would teach in the fall. It was too overwhelming to begin teaching the entire Core Knowledge program all at once, so the school phased it in-about half the first year, three-quarters the second year. Now the school aims to teach the entire program. The process of working to master a rich, content-oriented curriculum brought the teachers together as a team, Lewis said. "They were good teachers, but we were all isolated." The first day of the summer institute, Lewis said, "was group therapy. As an educator, what are your strengths, weaknesses, goals? They had never talked before."

The seventh-grade class of 2006-the class that became interested in Shakespeare's attitude toward women-was the first class to receive the benefit of the school's curricular improvements throughout its schooling. Four years before, Lewis said, 60 percent of the children were failing in third grade-"they were six months behind where they needed to be to be promoted." But by seventh grade, she said, they had written 10-page papers on such subjects as Sudan, Nazism, and the hardships faced by immigrants to America, and "will debate you on democracy and imperialism. They've really grown." Because of Core Knowledge, Lewis said, students "are really thinking critically. But it took seven years." She added that "everybody's looking for a quick fix," but real improvement takes time.

One of the jobs the school took on was to educate parents about the curriculum, in part because many of the parents didn't know the material and were upset that they couldn't talk with their children about what they were learning in school. "Teachers became teachers of the parents," Lewis said. All parents now
receive a copy of E. D. Hirsch's book, What Your First Grader Needs to Know: Fundamentals of a Good First-Grade Education, or the equivalent book for their children's grade level. Every six weeks, the school holds a Saturday workshop where parents learn about the curriculum and the tests their children are preparing for. While parents are in their classes, their children are off learning other material. In addition, there is a curriculum night every six weeks. There, parents learn about the curriculum in addition to learning how to help their children academically. "Some parents don't know how to color with children or how to read a book to their children," Lewis said. "So we teach them those skills." Before Core Knowledge was adopted, the school only attracted 10 or 12 parents to meetings, Lewis said; now, hundreds attend the workshops.

Lewis said that students at P.S. 124 bring to school all the issues of any large school. "We have lots of kids who have been hospitalized, who are suicidal, bipolar, schizophrenic, ADHD." The school provides a support system when things don't go well, providing referrals to social workers, health services, and housing services in addition to having a counselor, a half-time social worker, and a half-time school psychologist on staff. "We're a total-care facility," Lewis said, only half joking. "We get them bereavement groups, AA, drug rehab."

## Content Rich

In general, New York City is considered to have more of a skill-based curriculum than a content-based curriculum. Through the content provided by Core Knowledge, P.S. 124 works hard to make sure students learn the skills New York City wants taught. "Core Knowledge has really given us a focus. It really gives teachers the meat. But teachers still need to teach the skills," said Judy Lefante, the school's Core Knowledge coordinator. "You can't have one without the other, but we've worked hard through professional development to make sure they teach skills through
content." So, for example, skills such as making inferences, drawing conclusions, and separating facts from opinion are all worked on within the science and social studies content areas. In addition, Lefante said, "We try to integrate everything as much as possible so we don't have fragmented learning and children really build their background knowledge." If the children are studying Europe during the medieval period, for example, they read Robin Hood as well as nonfiction, Lefante said.

Lewis and assistant principal Linda Molloy are continually in classrooms, observing instruction and making sure that teachers and students are on track. "They want to do a good job," Lewis said. "My belief is that new teachers need time to grow." She has two or three teachers she considers marginal, so she sends in the literacy coach, the math coach, and the Core Knowledge facilitator to teach model lessons and help the teachers develop their skills. In addition, she said, she sends those marginal teachers into the classrooms of stronger teachers, arranges for professional development, and celebrates improvements. "The community needs to make each educator better," Lewis said.

To ensure that the school is on track, teachers and administrators monitor individual student growth on several measures, including unit tests. By studying the data, school staff members have identified the weakest area in the school to be grammar. Students often don't understand issues such as verb agreement and verb conjugation. To address the weakness, Lewis has purchased grammar textbooks and arranged for professional development for teachers on the subject.
"The expectations are always high," Lewis said. "It's about the belief."

Students appear to appreciate the expectations and the level of instruction. As one student, who came to P.S. 124 after being in another school, said, "I like this school better because you learn more things."
-K.C.

## Students at P.S. 124 in Queens Outperform Their Peers Citywide


(Continued from page 19)
the norm-referenced standardized tests most of us took growing up. But for the most part, those assessments were used as "summative assessments." That is, they were used to gauge what students knew, assign grades, and ultimately, sort kids into "high," "middle," and "low" reading or math groups in elementary school and tracks in secondary school.

Formative assessments are not designed to assign a grade but to gauge what students know about a particular topic or what they are able to do. In this way, teachers can understand where students are, what weaknesses or misunderstandings the students have, or whether they need additional enrichment or extension.

Some teachers may say, "We already have the state tests-we

> These schools have a respectful way of being honest about shortcomings. Failure merely means that studentsand teachers-have more work to do before they can be successful.

don't need more assessments." But that's not how the educators in It's Being Done schools think. They see state tests as useful end-of-year or midyear assessments that make sure schools and students are on track. But most state tests, for a variety of reasons, are not sufficient to guide day-to-day instruction. For one thing, results usually don't come back in anything under a couple of months. And, of course, most state tests are pretty low level. It's Being Done schools are aiming high, and they need to be able to see whether their students understand the material they are presenting and are meeting rigorous standards. For that, the schools need their own formative assessments. At Lockhart Junior High, teachers give quizzes in each core academic class once a weekstudents who score below 75 percent are immediately scheduled into "rescue classes" so that master teachers can figure out where the misunderstandings lie. At Graham Road, teachers go over every wrong test answer with every student so that they, too, can understand what led to the wrong answer. Sometimes it is just inattention; sometimes it is a misunderstanding of a word or a lack of background knowledge. In this way, teachers catch small problems before they grow.

It's Being Done schools also often use the formative tests as a way to ensure that their students are ready for both the format and the content of state tests. This is not the same as "teaching to the test." It is more along the lines of teaching students "test sophistication," as Valarie Lewis, principal of Osmond A. Church School in Queens, New York, calls it. Graham Road's Bensinger-Lacy is forthright about saying that children need help acculturating themselves to state tests. "I have no apologies for doing for our kids what middle-class families do for their kids. I'm hoping that when SATs come around, they'll understand how to take that kind of test." But the emphasis in all these schools is not on test-taking
strategies but on ensuring that students understand the material represented in high-level standards.

## Data-Driven Instruction

In It's Being Done schools, data are certainly used to identify which students need help and which need greater challenges. But there is another, more profound, way data are used as well: to see patterns that aren't always visible to teachers in their day-to-day teaching. So, for example, kindergarten teachers at Graham Road pore over color-coded charts to try to see patterns of achievement. In her first year, teacher Laura Robbins saw from the charts that in comparison with the students in other classes, her students didn't have many sight words. She asked her fellow teachers what they were doing to help their students. This is the kind of crucial interaction among teachers that has led to more students at Graham Road achieving at high levels than in most schools in Virginia.

Similarly, at Imperial High School, teachers spend a day before each school year looking for such patterns. One year they found that vocabulary was the weakest area for all groups of students-not just the English language learners. Once they identified that pattern, they were able to address the issue of vocabulary acquisition in a schoolwide way. Had the teachers simply been focused on their own students, they might never have noticed that even the high-est-achieving students in the school still had weaknesses in their vocabularies.

## Personal Relationship Building

It's hard for me to fully convey the atmosphere in It's Being Done schools and how different it is from ordinary schools. In essence, It's Being Done schools have an atmosphere of respect and caring that emanates from the teachers and principals. As Ware Elementary teacher Lisa Akard said, "We're a kind school. We really care about each other. The teachers care about the children." That caring is reciprocated by the students. So, for example, I could not find a student at Imperial High School who did not have good things to say about the school and his or her teachers. In comparing Imperial to his previous school, student Israel Ramos said, "The teachers there were just getting through the year-here they really care if you do your work and do well." Imperial's principal, Lisa Tabarez, expressed it this way: "It's not just about being successful in high school. We work for a greater accomplishment. We work for students to be successful, to take care of themselves and take part in society." Students respond powerfully to that commitment to their overall well-being.

When I say that It's Being Done schools are respectful, that doesn't mean that they put up with disruptive behavior on the
part of students-they do not. They do not let the learning of their students be disrupted for any reason, even another student. But they remain respectful, even of disruptive students. When John Capozzi, who is now principal of Elmont Memorial Junior-Senior High School, was assistant principal, he was in charge of discipline. His then-principal, Al Harper, said, "I've seen John suspend a student [and then] the student thanks him." That's how respectful the atmosphere is.

At Imperial High School, staff often have to explicitly train students, particularly new students, in the Imperial way of operating. "We start with where they are," assistant principal Aimee Queen says. One student, who had just transferred in and was completely unused to an orderly school, was given the initial goal of not get-

> Marginal readers in a special class were building genuine self-esteem based on the hard work of accomplishment.

I
able to teach; and, not incidentally, administrators who in ordinary schools would spend all their time on discipline are able to turn their attention to other issues, such as improving instruction. have described at some length the five elements of school reform as listed by Molly Bensinger-Lacy: teacher collaboration; a laserlike focus on what we want kids to learn; formative assessment to see if they learned it; data-driven instruction; and personal relationship building, all within the context of outside assessment.

There is something else that she didn't mention-something that I hope to explore more fully in future work-and that is leadership. Principals of It's Being Done schools set a vision for their schools and then helped teachers work toward it. And teachers set another version of that vision in their individual classrooms and then help their students work toward it.

All those leaders have embraced as a goal something that American public schools never before were asked to do: to educate all students to a meaningful standard. They all understand that to make that goal anything more than a pipe dream requires an enormous shift in how schools are organized and how they operate.
By making sure that everyone understands what children need to learn and then figuring out how to teach them, teachers and principals in It's Being Done schools have gone a long way toward devising the organizational structures that can help all students become educated citizens.

In contrast, the tradition of isolation that has characterized school organization has meant that too many children have gone to schools where there are no systems to ensure that they learn what they need. Affluent children, many of whom can draw on outside resources ranging from family dinner conversations to individual private tutoring, are often able to compensate for weaknesses in their school experiences. But children who live in poverty or isolation have fewer such resources to draw on, making them more dependent on schools and more dependent on educators figuring out how to ensure they learn.

It goes without saying that no school is perfect. Even the most successful have their mistakes, failures, and weaknesses. All have ways they can improve. This is, after all, difficult work requiring a lot of thought, skill, and effort-but educating all students can be done, and successful schools are showing us the way.

## Endnotes

1. Probably the best description of how schools are organized is by Harvard University's Richard F. Elmore, "Building a New Structure for School Leadership," American Educator 23 no. 4 (Winter 1999-2000): 6-13, 42-44, www.aft.org/pubs-reports/american_educator/ winter99-00/NewStructureWint99_00.pdf.
2. On teacher preparation programs, see, for example, the indictment by Art Levine (former president of Teachers College at Columbia University) of just about all such programs in Educating School Teachers (Washington, DC: Education Schools Project, 2006), www. edschools.org/teacher_report.htm. On colleague support, see, for example, Richard Kahlenberg's description of Albert Shanker's first year as a teacher in Tough Liberal: Albert Shanker and the Battles Over Schools, Unions, Race, and Democracy (New York: Columbia University Press, 2007). On teaching standards, see, for example, "There's a Hole in State Standards: And New Teachers Like Me Are Falling Through," by an anonymous second-year teacher, American Educator 32, no. 1 (Spring 2008): 6-7, www.aft.org/pubs-reports/ american_educator/issues/spring2008/newteacher.htm.
3. For some insight into the disconnect between teacher hopes and reality, see "Pursuing a Sense of Success: New Teachers Explain Their Career Decisions," American Education Research Journal 40, no. 3 (2003), which contains the results of a survey of 50 Massachusetts teachers.

## Coaxing the Soul of America Back to Life

 How the New Deal Sustained, and Was Sustained by, Artists

The United States of America was in crisis as 1934 approached. Art seemed irrelevant as the national economy fell into a profound depression after the stock market crash of October 1929. Thousands of banks failed, wiping out the life savings of millions of families. Farmers battled drought, erosion, and declining food prices. Businesses struggled or collapsed. A quarter of the workforce was unemployed, while an equal number worked reduced hours. More and more people were homeless and hungry. Nearly 10,000 unemployed artists faced destitution.

The nation looked expectantly to President Franklin Delano Roosevelt, who was inaugurated in March 1933. The new administration swiftly initiated a wide-ranging series of economic recovery programs called the New Deal. The president realized that Americans needed not only employ-
ment but also the inspiration art could provide. On December 8, 1933, the Advisory Committee to the Treasury on Fine Arts organized the Public Works of Art Project (PWAP). Within days, 16 regional committees were recruiting artists who eagerly set to work in all parts of America. Between December 1933 and June 1934, the PWAP hired 3,749 artists who created 15,663 paintings, murals, sculptures, prints, drawings, and craft works.

The PWAP suggested "the American Scene" as appropriate subject matter, but allowed artists to interpret this idea freely. PWAP images vividly captured the realities and ideals of Depression-era America. The PWAP art displayed in schools, libraries, post offices, museums, and government buildings lifted the spirits of Americans all over the country.


are protection of the family and the home, the establishment of a democracy of opportunity, and aid to those overtaken by disaster. Artists were among the helped, and thus were mobilized among the helping. Their paintings emerged from an otherwise dispirited nation as it sought an enhanced sense of itself, its common heritage, its common possibilities, and the common ground it occupied. ${ }^{2}$

Art came as the response of creative citizens to a challenge issued to them by their government. It offered those who needed it a meager living and in return they fulfilled a prediction made by the sculptor Gutzon Borglum to Harry Hopkins, Roosevelt's relief administrator. "Aid to the creative ones among us," wrote Borglum, would "enliven the Nation's mind" and help "coax the soul of America back to life." ${ }^{3}$

The New Deal was built upon the precept that the pursuit of happiness of each citizen was only possible in freedom from want, fear, hunger, and hopelessness. When the Roosevelt administration took office in 1933, its first order of business for the arts administrator, as for those administering programs across the country at large, was to make good on Herbert Hoover's assertion that "no one starved." Next, it sought "to put people to work." The national unemployment rate, which had been 3.2 percent of the workforce in 1929, became 25 percent in 1933-13 million people out of work-and did not fall below 10 percent until 1942. In 1934, there were many places in the nation, in cities and in the countryside, where half the willing workforce could find no jobs. ${ }^{4}$ In the 1920s, the affluent had danced the Charleston as the riffs of the Jazz Age mocked the miseries of the poor. Rural people had been afflicted for six years by crop failures, natural disasters, and falling farm prices. In 1933, the farmers had already been trying over and over again for seven years to get up and to stay on their feet, with dust and blood in their eyes, through the collapse of markets, insect plagues, blizzard, and drought.

The industrial system built upon the automobile industry had convulsed and collapsed; long before the stock
 market crash, fewer and fewer people bought cars. The industrial system went into its early convulsions as demand fell off for steel, rubber, copper, electrical products, and machine tools. Then an international banking and credit structure high on speculation became dysfunctional. Extravagance and imprudence no longer exhilarated corporate headquarters. The stock market crashed not once but thrice: in 1929, in 1933, and again in 1937.

The 1929 crash shook all expectations. The second and third shattered them. Some major industrial stocks lost four-fifths or ninetenths of their bubble prices and some never fully recovered. And where it mattered, in the real, tangible economy, nothing seemed to function properly. Millions were out of work, out of food, out of hope. None of the customary systems of society functioned in the


each other as they did during the wave of social cannibalism that gripped China and Russia, nor did farmers and farm laborers join doomed but destructive insurrections. Although there was a communist menace and there was a fascist menace, democratic government was sustained. The circle was not broken. The community held together-barely. Because the result was so uncertain, and because it often seemed as if the people could rely upon little more than Roosevelt's positive energy, an assertion of life in art-a demonstration of energy through creativity-mattered. The content of that art mattered as well. Often paintings such as these told us who we were, who we had been, and who we might become.

## Endnotes

1. Speech before the 1936 Democratic National Convention, Philadelphia, in Franklin Delano Roosevelt, The Public Papers and Addresses of Franklin D. Roosevelt, ed. Samuel I. Rosenman, vol. 5, The People Approve, 1936 (New York: Random House, 1938), 233. (On the Web, see www2.austincc.edu/lpatrick/his2341/fdr36acceptancespeech htm.)
2. For government's obligations, see Robert E. Sherwood, Roosevelt and Hopkins: An Intimate History (New York: Harper and Row, 1948), 59.
3. Gutzon Borglum to Harry Hopkins, quoted in Sherwood, Roosevelt and Hopkins, 59
4. For Herbert Hoover on "no one starved," see David M. Kennedy, Freedom from Fear: The American People in Depression and War, 1929-1945 (Oxford and New York: Oxford University Press, 1999), 86; and for "put people to work," see FDR's first inaugural address, March 1933, Washington, D.C., in Roosevelt, Public Papers and Addresses, vol. 2, The Year of Crisis, 1933, 13.


By Jennifer Dubin

When Sue Tabor stood before 20 fourth-graders at Pine Trail Elementary School one morning in April, they quickly forgot about the video camera and the 14 educators in the back of the room. They focused instead on Tabor, who said she was going to work with them on "a special math challenge." Tabor explained that after school the previous week, some teachers had played one of the students' favorite video games: Guitar Hero. The game, as the students already knew, entails playing a "guitar" to the notes of a rock song as they appear onscreen. If a player strums enough notes correctly, she "passes" the song and moves on to the next one. If she makes too many mistakes, she loses and the game ends.

Upon hearing that the teachers had played the game, the students' eyes grew wide and they giggled. "Now we know what those teachers do on break!" one student said. Tabor told them that the principal had not watched the teachers play and that she wanted the students to rank them so she could award prizes. Since the teachers had not played the same number of games, the students would have to figure out each teacher's rank. "You think you guys can help us?" Tabor asked. The students smiled and said yes. The teachers in the back of the room smiled, too; the lesson they had written was off to a good start. Tabor knew it by heart, and as soon as she mentioned Guitar Hero, she had the students hooked.

For three months, Tabor and other teachers at the school in

Volusia County, Florida, had worked on this particular lesson, an introduction to percentages. They reviewed their state's standards and researched ways to teach proportional relationships. They created a blog where they posted comments as the lesson developed. They consulted math education experts. Meeting during school and on in-service days, they carefully chose which words to use in discussing the mathematics they wanted to teach and which numbers to use in creating problems. After Tabor taught the lesson, the teachers discussed it at length and then one of them wrote a summary of their reflections. They took these steps to craft a single lesson, a practice they engage in once a year. This complex process has a simple and meaningful name: lesson study.

## Teacher-Led Professional Development

In Japan, jugyou kenkyuu-or lesson study-is the most common form of professional development among elementary school and lower-secondary school (grades 7, 8, and 9) teachers. While in the United States it is best known as a means of improving math instruction, in Japan lesson study is practiced in all subjects, from language studies to physical education. Teachers typically begin engaging in lesson study as part of their pre-service training and then continue the practice throughout their careers.

Teachers (sometimes in the same grade, sometimes across grades) meet regularly over several months to plan what is called a research lesson. First, they decide what concept to present to students. Then, they consult books and articles that other teachers have written. Such resources are available because lesson study groups write reports after their lessons, and those reports are often published and sold in local bookstores. Japan's national curriculum makes this exchange of ideas fairly easy; for instance, fifth-graders learn the same material no matter which school they attend.

In developing the lesson, teachers try to agree on every detail, even the exact phrasing the teacher will use in explaining key concepts. They also anticipate students' responses so they can plan how the lesson will unfold and be prepared to address students' mistakes. Just as important, teachers focus on hatsumon-posing key questions to stimulate students' thinking. With the

[^11]right questions, teachers can guide students to a better understanding of the problem at hand, and how it relates to previously learned material.

As needed, teachers draw on outside experts, known as "knowledgeable others," to assist in planning and to observe and comment on the research lesson. These experts include college professors who specialize in the relevant content area or in cognitive science, accomplished teachers from schools that work closely with national universities, and instructional supervisors. Knowledgeable others often work with many lesson study groups throughout the school year, enabling them to contribute not only their own content and pedagogical knowledge,
 but that of multiple lesson study groups. Since these experts observe research lessons frequently, they see examples of excellence and push all their groups to improve.

Early in the planning, teachers set a date for the lesson and choose someone in the group to teach it. While that person is teaching the lesson on the scheduled day, the other teachers in the group observe and take notes on student responses. Often, a video camera records the lesson for the teachers to review.

After the lesson has been taught, teachers often spend 60 to 90 minutes discussing it. The teacher who taught speaks first. She tells the group what parts of the lesson worked as planned and what could improve. Then the other teachers share their observations. It's important to note that the teachers focus their comments on student learning during the lesson, which they all planned, not on the teacher who taught it. Lesson study is not a tool for teacher evaluation. Members of a lesson study group seek to improve their students' understanding of concepts and, in the process, work together to improve their teaching.

Based on their reflections, the teachers revise the lesson, and then another member of the group teaches it to another class. This time, other teachers (those in the school and elsewhere) plus outside experts often are invited to observe the lesson and participate in the postlesson discussion. Again, the teacher who taught the lesson shares her insights first. Usually, a moderator focuses the discussion so observers can share their thoughts on what students learned during the lesson. At the end of the discussion, an outside expert usually makes closing remarks. Finally, members of the group write a report summarizing their work.

The goal of lesson study is not to create lessons, though that is one benefit. The goal is to engage teachers in a research process that will help them improve their teaching. Lesson study provides a framework for Japanese teachers to think deeply about content and student learning. It also gives them an opportunity to learn from each other. This contrasts sharply with the isolation that so often characterizes teaching in America. Here, teachers have little time to exchange ideas for improving instruction and rarely
observe each other.
Of course, the process is not perfect. A common criticism of lesson study (especially as it is practiced in the United States) is that if teachers do not have sufficient content knowledge, their efforts may not be productive. One obvious way to improve the lesson study process: draw on experts from the outset, particularly when trying to address a concept that teachers and students alike find challenging.

Teachers in the United States may need to call on "knowledgeable others" even more often than their peers in Japan. As Catherine Lewis, a lesson study researcher at Mills College, has pointed out, U.S. teachers do not have a rich national curriculum, topnotch textbooks and other instructional materials, informative teachers' manuals, or a long history of practicing lesson study. Japanese teachers have all these things, plus even more supports (like highly focused teacher preparation), which better prepare them to undertake lesson study.

Of all the supports that U.S. teachers lack, the absence of a concise, coherent, common curriculum may be the most problematic. Here's how Patsy Wang-Iverson, a lesson study researcher, put it:*

In Japan, lesson study is perhaps more viable because the curriculum is focused on fewer topics than typical U.S. curricula. For the sake of comparison, consider that a science topic such as pendulums might require 13 to 14 lessons in Japan. . . . During these lessons, students have the opportunity to (1) decide what variables they need to investigate, (2) design and conduct the experiments, and (3) frequently repeat their experiments to test the validity of their findings. . . . In the United States, that same topic may be covered in one class period to make time for other required

[^12]topics. Under which circumstance do we think students will develop a deeper understanding of pendulums?

Not surprisingly, Wang-Iverson suggests that schools address their overstuffed curricula before undertaking lesson study.

## Lesson Study Comes to Volusia County

In the late 1990s, a groundbreaking international video study* of eighth-grade classroom instruction brought to light dramatic differences between the United States and Japan. Researchers found that Japanese teachers often focused their math lessons on developing students' understanding of the relationships between mathematical concepts, while American teachers often focused more on procedures and skills. Although the

> Teachers in the U.S. do not have a rich national curriculum, topnotch textbooks, informative teachers' manuals, or a long history of practicing lesson study. Japanese teachers have all these things.

video study could not determine what caused these differences in instruction, some of the key factors appeared to be Japan's national curriculum, high-quality instructional materials, and commitment to lesson study.

Since 1995, a handful of math education experts in the United States have worked with teachers to form lesson study groups. The one who brought lesson study to Volusia County is Alice Gill. A former elementary teacher, Gill now develops and coordinates math professional development courses for the American Federation of Teachers.

In January 2003, Gill gave a presentation on lesson study at the Volusia Teachers Organization (VTO) ${ }^{\dagger}$ office. Soon thereafter, a group of eight intermediate-grades teachers from six different schools-including Pine Trail Elementary-began meeting regularly. The group conducted its first research lesson (on the distributive property) in March 2003. After working as a multischool team for three years, and developing enough research lessons to become comfortable with the process, members of the group decided they'd like to develop lesson study groups in each of their schools.

Becky Pittard, a member of the VTO and a fourth- and fifthgrade teacher at Pine Trail, eagerly brought the practice to her

[^13]school. Now enough Pine Trail teachers express interest to form at least one and sometimes two or three lesson study groups in math, science, and writing each year. Before lesson study, teachers didn't really collaborate on improving instruction. As they passed each other in the halls, they might share ideas, but they didn't have a dedicated block of time to discuss content, student learning, or instructional strategies.

One Friday morning in January 2009, ${ }^{\ddagger}$ on a teacher professional development day, Pittard and her colleagues did have that time. A lesson study group that focused on writing met in one classroom, while in Pittard's classroom, the lesson study group that focused on math began discussing, in
person, its research lesson.
The members of the math group, composed of teachers in kindergarten through fifth grade, had brought books and research articles to Pittard's classroom to help them brainstorm. A few weeks earlier, they had started to share ideas on a blog they had created. Pittard, the math group's facilitator, reminded the teachers that, as they had already discussed on their blog, the upper-grades teachers wanted help teaching percentages,
"a very difficult concept for children."
Pittard was concerned because the topic too often has been taught not for understanding but solely for doing the operation.

The teachers scanned the piles of papers and books on their desks, including math textbooks from Singapore (which are written in English) and Japanese math textbooks, translated into English. The books are slender and colorful, with a small number of carefully sequenced topics per grade. They hardly resemble American math textbooks-tomes that cover too many topics and overwhelm students and teachers alike. They also flipped through another resource, Thinking Mathematics. Created jointly in 1992 by AFT teachers and staff, and cognitive scientists from the University of Pittsburgh, Thinking Mathematics is a program that teachers can use with any math curriculum. Thinking Mathematics includes research-based articles, instructional strategies, and content knowledge. Nearly all of the school's 46 teachers are trained in it.

As the group searched for a clear way to present the idea of percent, Stephanie Hajdin, a first-grade teacher, read aloud from one of the books from Singapore: "Percent is out of 100 or per 100 ." The teachers examined the Singaporean books further. They noted how the problems work out evenly so students can focus on understanding concepts and not be distracted by computation. They also admired the books' organization. When Hajdin pointed out that students first learn ratios, then fractions, then percentages, Pittard said it made sense. Students at Pine Trail and across the United States, she said, don't learn those concepts in that

[^14]order—but they should. Ultimately, they decided to craft a lesson on ratios.

At the end of the two-and-a-half-hour meeting, Pittard encouraged the teachers to continue sharing ideas on the blog. She said they would start shaping the lesson at their next session. The group met five more times before April 2, the date they had set for teaching the research lesson.

## Crafting and Teaching the Lesson

Three months later, the teachers had come a long way from their January meeting. They had settled on introducing a fourth-grade class to the concept of ratio through a story they made up about teachers playing Guitar Hero. Pittard and her colleagues had written the lesson to illustrate that if players don't play the same number of games, their scores must be calculated as ratios-comparing the number of songs played with the number of songs passed-to determine the winner. The scores from this popular video game captured the students' attention. Ultimately, the game provided the teachers with a hook to give students a concrete example of ratios.

Sue Tabor, a special education teacher who had participated in lesson study for the last three years but had not taught a research lesson, volunteered to teach. The group also had kept in touch with two "knowledgeable others." In addition to Gill, Tad Watanabe, a professor of mathematics at Kennesaw State University, had offered suggestions during the lesson's development.

Lesson study groups in the United States often do not follow the Japanese model to a tee. Circumstances force them to tweak the practice. For instance, at Pine Trail, after Tabor taught the official research lesson, the group did not have another member of the group teach it. They did revise it, and some teachers plan to use it in the future, but unfortunately they were unable to complete two observations, and postobservation discussions, of the lesson. Another difference was that the teachers at Pine Trail had limited access to outside experts. At times, they struggled with some of the concepts-which is to be expected, since their goal is to improve their teaching and students' learning of challenging content. It would have been helpful to have experts observe their research lesson and participate in the postlesson discussion.

On April 2, minutes before the research lesson began, Tabor walked into the classroom to a round of applause. The members of the group cheered along with the principal, assistant principal, and a teacher from another elementary school who would moderate the postlesson discussion. Tabor admitted she was nervous. Pittard told the group that Tabor had nothing to be nervous about. "Our observations need to be focused on the behavior of the children, not the teacher," she said. Pittard reminded her berore theresearchleswalked into the class -
colleagues to stay focused on student learning during the lesson.

The observers each took a copy of the research lesson, which was divided into three columns: one for what the teacher says in each step of the lesson, one for anticipated student responses, and one for what the teacher says when a student's work is not on target. Each observer received a clipboard for taking notes, and a classroom seating chart. Pittard reminded everyone not to talk during the lesson, but invited them to walk around the room to hear the students' conversations once group work began.

The students entered the room. Their classroom teacher, who had agreed to the students' participation in the lesson, gave them nametags so the observers could match names with faces and comments. Those comments would help them understand the lesson's effectiveness.

As planned, Tabor began the lesson by explaining that the principal had asked the students to rank the teachers. Then she launched into the group's introduction: "Sometimes, when we solve math problems, we have to do a lot of work with adding, multiplying, or dividing numbers. But sometimes, mathematicians look at a problem and just use their common sense." Tabor then posted a question on the board: "How can we make it easy to compare scores?" She showed the class the first set of players and scores:

Mrs. Hajdin passed 4 out of 10 songs.
Mrs. Maccio passed 2 out of 10 songs.
Mrs. Wachtel passed 4 out of 7 songs.
Tabor asked everyone to reflect on the scores and tell her what they noticed. Then she asked them to share their strategies for ranking the teachers. It appeared that half the students understood the necessary proportional thinking and were keeping the ratio of wins to games played the same. They correctly ranked Mrs.


Wachtel first, Mrs. Hajdin second, and Mrs. Maccio third. The other half of the students used subtraction: they said Mrs. Wachtel should be ranked first because her score-4 out of 7 -is a loss of only 3 games, while Mrs. Hajdin's score-4 out of 10 -is a loss of 6 games.

Both approaches led students to the right answer. In planning the lesson, the teachers accurately predicted that some students would use subtraction, which, of course, will not always work. To explain why it does not work every time, the teachers wrote what Tabor should say. "If Rohit played 99 games and won 97, and if Julie had time to play 2 games and won 1, does that make her a better player?" Tabor asked. The students said no. Tabor called on Chase to explain why: Julie had won only half her games. Tabor reminded them to keep each teacher's ratio of wins to total games played the same as they compared scores in order to rank the teachers. Tabor presented two more sets of scores, neither of which resulted in the correct ranking if students used subtraction. Throughout the lesson, Tabor walked around the classroom to answer students' questions. The observers walked around, too. They listened to students' conversations and took notes.

## The Postlesson Discussion

After the lesson, Tabor and the observers gathered in the school's media center. Tabor spoke first. She said she was glad she overcame her fear of teaching before her peers and that the students seemed to get the goal.

The teachers congratulated Tabor on teaching the lesson, and themselves for successfully anticipating students' responses, particularly their misunderstandings. For more than an hour, the teachers worked to improve the lesson. They wanted to add different phrases and emphasize certain words to make the lesson more effective in reaching all students the next time it was taught. At Pine Trail, research lessons don't sit untouched on a shelf. Teachers use them in their own classrooms long after they are written.

To improve the lesson further, Pittard e-mailed a summary of reflections to the "knowledgeable others," Watanabe and Gill, and asked what worked and what could improve. In Japan, knowledgeable others usually attend lessons and participate in the postlesson discussions. Ideally, they would do the same in this country. When that is not possible, reflections by e-mail are worth gathering. Gill was pleased that the lesson required the students "to draw on what they already knew to compare the scores, instead of just giving them a formula to use to make the comparison."

As for something to improve, Watanabe suggested that the teachers avoid using the term "rate" and only use the term "ratio." In this lesson, both terms were used interchangeably, something he says happens often because there are no set definitions. He finds the following definitions helpful: "A ratio is a comparison of two (or more) quantities of the same kind, while a rate is a comparison of two different quantities." Having not observed the lesson, he can't say for sure, but it's possible that some students assumed that when pretending a teacher played more games than
she did, they had to keep her "pace" of winning the same. Indeed, one student who struggled with the lesson did seem to be thinking along those lines. He commented to another student that one teacher, who had played 3 games and won 1, would win once every time she played 3 games.

Overall, Watanabe found the core idea of the lesson quite strong, saying "the essence of putting ratios in the context of making multiplicative comparisons is something that other lesson study teams should think deeply about."

## Principal Support

Lesson study at Pine Trail, or at any school, would not happen without the principal. When Pittard first approached Barbara
> "I'd rather do this any day of the week than attend a workshop for three hours and have somebody tell me what I should be doing in my classroom."

-STEPHANIE HAJDIN,
First-grade teacher

Paranzino, Pine Trail's principal for 16 years, Paranzino was skeptical: "I really thought it was so time consuming and that there would be no way we could pull this off." Gradually, she saw that lesson study was time well spent, that the purpose was not to create the perfect lesson. "We're after the growth," she says. "Teachers communicating with each other about a specific math concept-that conversation is an administrator's dream."

To make it a reality, at the beginning of each year she and Pittard ask teachers if they want to participate in lesson study. After the groups form, she and Pittard schedule dates for teaching each of the research lessons. They also work around school vacations and state testing to schedule blocks of time-typically 60 to 90 minutes-for the groups to meet. The days the research lessons are taught, Paranzino helps ensure that teachers not involved in lesson study can cover the classes of those who do participate. Some years, the school uses grant money to pay for substitutes.

In an effort to drum up support for lesson study districtwide, Paranzino has invited other principals to observe research lessons at Pine Trail. But she emphasizes that interest in the practice must come from teachers, not from the top down. "It's a huge commitment." Unfortunately, in Volusia County, teachers engage in lesson study without extra pay and often on their own time.

For Stephanie Hajdin, a first-grade teacher, the practice tops all other kinds of professional development. "I'd rather do this any day of the week than attend a workshop for three hours and have somebody tell me what I should be doing in my classroom," she says. Instead, Hajdin and her colleagues decide what they need to work on. Each year, when she signs up for lesson study, she looks forward to improving her teaching, to sharing in the camaraderie and the sense of accomplishment. "I really enjoy being part of the team."

## Learning Science Content-With Reason

By Paul R. Gross

If you are an experienced and loving teacher, you probably have felt the mixed pleasure and pain brought on by students' struggles to display their content knowledge and ability to reason. Surely, you've seen more than a few exam answers like these: ${ }^{1}$

Nero was a cruel tyranny who tortured his subjects by playing the fiddle to them.

The sun never set on the British Empire because the British Empire is in the East and the sun sets in the West.

Gravity was invented by Issac Walton. It is chiefly noticeable in the autumn when the apples are falling off the trees.

Such answers tickle us because of the mismatch between the test-takers' logic and sentence structure-both of which are normal-and one or more preposterous details of their assertions. The faulty detail can be as simple as a misspelled or misused word, or as flagrant as complete failure to relate cause to effect. Clearly, ordinary competence in language and logic are not enough to keep us from coming up with howlers-if we don't know, or we simply misunderstand, important details of a subject we address.

This is as true in science education as elsewhere in life. And so, in the course of a long career as a biologist and teacher of science, I have often been troubled by the endless debate about whether we should

[^15]
of Sputnik, could conclude that just memorizable facts were required, with no reasoning. Facts were (and are) taught, and facts must be learned if any intellectual discipline is to be understood and practiced. The rhetorical flourishes of those arguing for more scientific reasoning have affected some people's perceptions, but they have not changed the reality that, in general, science curricula have never been exclusively
lists of facts to be memorized, devoid of the means by which those facts are discovered and gain acceptance in the scientific community.

Before we go any further then, let's pause for a moment to consider just what scientific reasoning is. What differentiates scientific from, say, historical reasoning? Other than the content being reasoned about, I can't think of anything. So, I turn to the distinguished philosopher of science and epistemologist Susan Haack to discover that the notion of a species of reasoning unique to science is unfounded. Haack writes: ${ }^{2}$

Scientific inquiry is continuous with the most ordinary of everyday empirical inquiry. There is no mode of inference, no "scientific method," exclusive to the sciences and guaranteed to produce true, more nearly true, or more empirically adequate results. . . . And, as far as [science] is a method, it is what historians or detectives or investigative journalists or the rest of us do when we really want to find something out: make an informed conjecture about the possible explanations of a puzzling phenomenon, check how it stands up to the best evidence we can get, and then use our judgment whether to accept it, more or less tentatively, or modify, refine, or replace it.

The practices of good science are distinguished by that "informed conjecture"-by a special dependence upon technology (e.g., instruments that broaden the human range of perception), and by especially strong and well-enforced rules having to do with scrutiny and testing of claims and reproducibility of results. But they are not distinguished by an array of clearly identifiable, cognitively unique forms of reasoning.

What, then, is to be understood by scientific reasoning? The answer cannot be very deep because the question isn't. Scientific reasoning is using, within a framework of scientific content, certain general cognitive abilities that develop over time or can be encouraged in most learners. So, there is not much that is exclusively scientific about such reasoning other than the fact that one is thinking about scientific content. Scientific reasoning is a sibling to, if not perfectly congruent with, historical
reasoning, which is the use of rather similar cognitive basics in the context of records and commentary on the past. Scientific reasoning is deployed with hypotheses and observations about nature. It has other siblings as well: social, artistic, and literary reasoning for example.

For those concerned with school science, however, the issue is scientific reasoning, and the goal is to encourage bet-ter-informed rationality about nature, to bring about significant improvements in students' scientific literacy and problem-solving skills. Of course, there is an enormous literature on the question of how to do this. At least among cognitive scientists, the consensus seems to be that, "Just as it makes no sense to try to teach factual content without giving students opportunities to practice using it, it also makes no sense to try to teach critical thinking devoid of factual content." ${ }^{3}$ Here, for "critical thinking," we may substitute "scientific reasoning." In the relevant contexts, they mean almost the same thing: scientific reasoning in the absence of scientific content doesn't make sense. Reasoning and content are not practically and neatly separable.

So, why isn't this old debate over? Why, in fact, is there a debate at all? Unfortunately, it seems that ongoing, important, and often laudable research on how to increase students' science learning continues to stumble, from time to time, over these questions. This is understandable: any researcher will tell you that gathering data about complex processes is the easy part; making sense of those data, and drawing sound conclusions from them, is the hard part. So it's important that all of us, not just researchers but teachers too, question studies that reach puzzling conclusions. Not because we, individually, will thereby come up with the "right" conclusion, but because such questioning is essential to ensuring that the research enterprise as a whole advances both intellectually and in its eventual usefulness.

## Scientific Reasoning in Science Magazine

Let's examine a recent article on scientific content and scientific reasoning that has received a good bit of coverage in the popular media. A few months ago, Science-one of the two most selective international science journals (the other one is Nature) -


LAVA, PHOTO BY J.D. GRIGGS, 02/25/83, JG928, COURTESY OF THE U.S. GEOLOGICAL SURVEY
published an important article on a study of learning and scientific reasoning. ${ }^{4}$ This fascinating paper has some perplexing features. Science's summary of the study declares that "comparisons of Chinese and U.S. students show that content knowledge and reasoning skills diverge." Now, such a showing ought not be in the least surprising to the journal's readers. "Divergence" is both innocuous and ambiguous; and as we have suggested, the claim that content and reasoning can be separated has been afloat for many years.

Nevertheless, however commonplace the statement, such a divergence would be very important if it were (1) anything more than a simple acknowledgement that content knowledge and basic reasoning skills are in some respects different things, and (2) demonstrated unequivocally to exist, with rigor typical of most Science articles. It would be very important not only for K-12 science, but for all education. But as noted, the article, titled "Learning and Scientific Reasoning," offers some puzzles. They need to be considered before the study's conclusions are taken as grounds for action. Among the firmest-and yet most questionable-conclusions offered in the text is this: ${ }^{5}$


The current style of content-rich STEM [science, technology, engineering, and mathematics] education, even when carried out on a rigorous level, has little impact on the development of students' scientific reasoning abilities. It seems that it is not what we teach, but rather how we teach, that makes a difference in student learning of higher-order abilities in scientific reasoning.

To restate: "Higher-order scientific reasoning" cannot be achieved by science learners if they are offered only "contentrich" science courses and programs. Something different must be added or substituted. That something, according to the authors, is the explicit teaching of scientific reasoning, here (as commonly elsewhere) identified with inquiry learning. Within the enforced economies and terseness of the
instructional time and the amount of emphasis on conceptual physics understanding and problem-solving skills are very different in the two countries." This, they claim, provides what is, in effect, a controlled experiment, an opportunity to see if these variations in content learning-intensive, as in China, versus (relatively) superficial, as in the United States-have an impact on scientific reasoning ability.

Here, however, the first puzzle of the study appears. The description of content learning in the United States indicates correctly that it is less intense and
come from three tests-two of physics knowledge and one of general scientific reasoning-administered to freshmen college students in the United States and China. All the students were science or engineering majors, enrolling in college-level, calculusbased physics-but the tests were given before instruction began. The authors, Lei Bao and a dozen colleagues, specify carefully the differences between these two cohorts. The most striking is their precollege preparation in physics. Bao et al. explain that "Chinese students go through rigorous problem-solving instruction in all STEM subject areas throughout most of their K-12 school years and become skillful at solving school years and become skillful at solving
content-based problems." This is, as they note, in sharp contrast with K-12 science education for U.S. students, who probably spend less time in science study of any kind and, obviously, less time doing physics. As the authors observe, "The amount of
writing characteristic of Science and Nature, a claim such as that is usually taken very seriously. Should this one be so taken? To find out, we must examine the data provided and (using scientific reasoning and relevant content from cognitive science) judge the conclusions drawn from them.

Data for this study
more varied than in China. But then it claims incorrectly that "scientific reasoning is not explicitly taught in schools in either country."

Had this paper, with its generous online supplementation and other publications from the lead author's research group, failed to show awareness of the current research literature in K-12 science education, their claim that scientific reasoning is not being taught would have been understandable. And, thus understood by us, the study would simply have been ... dismissible. Why? Because it is not true that scientific reasoning is not taught in U.S. schools.

Scientific reasoning goes by different names, one of the most favored being "inquiry," as in "inquiry-based learning." This type of science study is so well established in the United States that a booklength retrospective and prospective account of inquiry-based science standards was published by the U.S. National Research Council nearly a decade ago. ${ }^{6}$ One need only skim the most recent Fordham Institute study on state science standards to discover that scientific reasoning and "science process" skills, which focus on reasoning, are key elements of the expectations for student proficiency in nearly all of the 50 state standards reviewed. ${ }^{7}$ The current Science Framework

[^16]
for the National Assessment of Educational Progress reflects that preoccupation by dividing attention between science content and science practices. Of the latter, there are four, each preceded by an action verb: "identifying" or "using." The "using" statements are explicit reasoning skills. ${ }^{8}$

However, Bao et al.'s own print and online bibliographies do cite appropriate contemporary resources, indicating that they have at least come in contact with the evidence of inquiry-based learning in U.S. science classrooms. Hence, their statement that scientific reasoning is not taught in U.S. schools is not due to ignorance. It is just a misconception of current, standardsbased science curricula nationwide, and of the associated literature.

Now the second puzzler appears. Although they refer to the physics courses, especially those taken by the Chinese students, as emphasizing "conceptual physics understanding and problem-solving skills," the researchers do not, apparently, include conceptual understanding and problemsolving skills within scientific reasoning ability. For this old subscriber to Science, such an exclusion is incomprehensible.

These lapses are regrettable because they create a flaw in the experimental design, the clarity of which depends upon the assumption that neither American nor Chinese K-12 science students receive special instruction in scientific reasoning. In reality, all available evidence indicates that both U.S. and Chinese students receive at least some instruction in scientific reasoning.

The authors believe that in these otherwise matched student groups, the students have a clear, large difference in exposure to

and study of physics content. If this is so (and there is no reason to doubt it), and assuming that a good test of scientific reasoning not tied to content-i.e., not domain specific-is available, then it is possible to test for the impact of that difference in studying content on scientific reasoning ability. More specifically, it is possible to test for that holy grail of instruction, transferability. Transferability would mean that students become good scientific thinkers generally-that their reasoning transfers smoothly across all scientific subjectsinstead of being limited to the specific areas they have studied. In the authors' words, they are interested in "domaingeneral reasoning skills such as the abilities to systematically explore a problem, to formulate and test hypotheses, to manipulate and isolate variables, and to observe and evaluate the consequences." So the ultimate question that Bao et al. undertake to answer is whether the Chinese students, recipients of prolonged and intense content instruction, are rendered thereby more adept at general scientific reasoning than the Americans, whose study of physics and other science content has been slight by comparison.

## Performance in Physics and in Scientific Reasoning

To answer this question, Bao et al. employed three good tests: the Force Concept Inventory (FCI), ${ }^{9}$ which assesses knowledge of introductory Newtonian mechanics; the Brief Electricity and Magnetism Assessment (BEMA), which assesses understanding of electricity (including circuits) and magnetism; and the Lawson Classroom Test of Scientific Reasoning (LCTSR), which is supposed to assess capacity for general scientific reasoning (that is, with minimal domain dependence). To the authors' credit, the

Science print article and its online supplements together provide adequate detail on the tests, the testing, and their results.

Outcomes of the tests are clear enough in the article and supplements. On the FCI, Chinese students performed very well, with a narrow distribution of scores centered on an impressive mean of 86 percent. The American scores were much more broadly distributed around
a mean of 49 percent-distinctly failing. On the BEMA, Chinese students scored at a mean of 66 percent, but the Americans scored at a mean of 27 percent-not much better, Bao et al. note, than would have been produced by randomly choosing answers to the test questions. These tests distinguished the two populations of test takers, one well prepared in physics, the other not.*

So far, no surprises. These results look like those of recent international assessments in science and mathematics, in which the performance of U.S. students, especially in the higher grades, is at best undistinguished and sometimes awful.

The results of testing scientific reasoning with the LCTSR, however, were surprising (to me). Both groups showed a mean of 74 percent and their score distributions were effectively identical. ${ }^{+}$Such results should be surprising, at least to many Science readers; but the authors, instead of being surprised and questioning the results, conclude that they have a substantive finding regarding scientific reasoning,

[^17]worthy of their most important comments and recommendations. They believe the results indicate that content instruction, in physics anyway, cannot inculcate good scientific reasoning abilities and habits. More study of content leads only to more "content knowledge," not to that higherlevel, general competence in science that is so eagerly sought.

I am sorry that the authors were not surprised by their findings. Had they been surprised, they might have questioned their immediate response to the data and considered alternative conclusions. The job of considering alternatives, then, is left to others.

## What Do the Scores Mean?

Let's set aside, for the moment, our earlier concern about whether U.S. and Chinese students are actually taught scientific reasoning in an explicit way, and take the information presented by the authors at face value. It indicates that the training Chinese students receive before coming to college includes much practice with important concepts of physics and with skills needed to solve physics problems. Tested at the end of this period for knowledge of two central physics topics, the Chinese students perform handsomely. Not only are they ready for calculus-based college physics, but they can be said, in all justice, to know physics, at least the physics taught in high school. For students in the United States, the situation is essentially the opposite. Only a third or so of them take high school physics. The rest learn physics, if at all, from the general science of grades K-8 and via the (derived) physics components of other science disciplines, such as biology, chemistry, Earth science, or environmental studies. These students perform poorly on the physics tests. They cannot be said to know physics.

Now, both cohorts are tested with the LCTSR for their ability to think about very simple natural (i.e., scientific) situations, for example: explaining the results of filling graduated cylinders of differing diameter

[^18]with the same volume of liquid, and vice versa. The test questions have mainly to do with logic and efficient thinking. On such a test, both cohorts perform at a solid average level; and what's more, the population score distributions are essentially the same. What is going on?

Bao et al. conclude that even though the Chinese students know physics content, their scientific reasoning is no better than that of the American students. As for scientific reasoning that is transferable and immediately usable in real-world problems, the authors evidently believe, Chinese students are no better equipped than those contentchallenged ${ }^{10}$ U.S. students.

But this is not a necessary, or even the most likely, conclusion. A more likely one is that the LCTSR is testing the students' reasoning about certain simple but unfamiliar natural situations. So, it requires all the test takers, Chinese and American, to rely on the same relatively slow, relatively inefficient kind of thinking.

The findings of cognitive science tell us that domain knowledge strongly affects the quality of thinking. Specifically, its accuracy, speed, and efficiency-manipulating information in working memory-are much improved when relevant, quickly recoverable knowledge (procedural as well as factual) is stored in long-term memory. So, if you want to solve physics problems quickly and efficiently, you'll need a good bit of factual and procedural physics knowledge stored in your long-term memory. How is such knowledge stored in long-term memory? By solving physics problems! Bit by bit, you tackle more and more complex problems, and eventually you have in long-term memory a rich domain of physics facts, procedures, and tricks of thought about concepts of physics and physicslike problems.

Faced, then, with a new problem in physics, you ordinarily will retrieve examples of correct solutions to similar problems encountered earlier-not the primitive steps of the required solution (which from practice have become automatic for you, like number facts in arithmetic). You will automatically import from long-term memory into working memory whole chunks of problem types and solutions, and thus will be able to grab quickly the appropriate one for application to the new problem. Your thinking (or reasoning) will be efficient. You will do well in a written or



SUPERCONDUCTIVITY, © LAWRENCE MANNING/CORBIS
a real-world test. ${ }^{11}$ Reasoning works with content!

Here, then, is an alternative view of the Bao et al. results. The Chinese students know physics. The American students don't. Now both groups face a different challenge-different enough from the standard physics problems so that the Chinese students' superior conceptual and problem-solving skills in physics provide no immediate advantage. The new challenge is to think about problems of a very simple scientific character, but in forms and subject-matter domains that neither group has encountered before. As the authors explain in their online supplementary materials, the LCTSR "measures fundamental reasoning components with simple context scenarios that do not require complex content understanding. This test design can improve the measurement of the basic reasoning abilities by
reducing the possible interference from understandings of content knowledge." But if so, both cohorts will handle most of the questions on the LCTSR (or any challenge like it) the same way: they will need to think through each question from scratch-to find the best answer starting from elementary principles. That kind of thinking is slower and more error-prone than the thinking available to a physics-savvy Chinese student taking the FCI or the BEMA.

There is one remote possibility to consider. Going back to the first puzzle, suppose that, contrary to a crucial assumption of the authors, the American students do receive considerable instruction in what they call scientific reasoning, and (as the authors claim) the Chinese students do not. That could, in principle, account for the Americans performing well enough to match the performance of the Chinese. But any such explanation seems extremely unlikely, given the remarkable congruence of the LCTSR results of both groups. And, if the Chinese students had really received no scientific reasoning instruction, we would expect the Americans, who have been taught scientific inquiry, to do much better on the LCTSR than the Chinese. They did not.

That, of course, raises the possibility hinted at by the second puzzle. It could be that both the U.S. and Chinese students receive instruction in scientific reasoning. Bao et al. may not define it that way, but an "emphasis on conceptual physics understanding and problem-solving skills," which is how they characterize the Chinese instruction, sounds to me like plenty of emphasis on reasoning about scienceand about much else! So it may be that both the Chinese intensive approach and the American nonintensive approach are equally effective-or equally ineffective-in teaching the domain-independent reason-
ing the LCTSR is supposed to test.
One final possibility is a rather unhappy one, but perhaps the most realistic. It could be that, because the students were matched in every relevant characteristic except physics "content" instruction, these two large student groups, Chinese and American, have simply reached the same level of general reasoning ability (or have the same average IQ). ${ }^{12}$ So the LCTSR, with its general reasoning questions, is simply establishing a good control (a proper isolation of variables) - that is, these two groups of rather well-matched students are of about the same general cognitive ability. And, of course, that would be comforting in some ways, but also no surprise.

Whichever conclusion(s) may be correct, what we can say with confidence is that these Chinese students learned enough physics in school. The U.S. stu-dents-who, having opted already for science, technology, engineering, and mathematics majors in college, are among our best science students-have not learned enough. That should be a big worry, and not only because, as we saw at the outset, reasoning devoid of content can prompt a chuckle or two.

## Endnotes

1. For these and other examples of students' misunderstandings (along with explanations), see www.britishcouncil. org/learnenglish-central-stories-exams.htm.
2. Susan Haack, Defending Science—Within Reason: Between Scientism and Cynicism (Amherst, NY: Prometheus Books, 2003), 24.
3. Daniel T. Willingham, "Critical Thinking: Why Is It So Hard to Teach?" American Educator 31, no. 2 (Summer 2007): 8-19, www.aft.org/pubs-reports/american_ educator/issues/ summer07/Crit_Thinking.pdf.
4. Lei Bao et al., "Learning and Scientific Reasoning," Science 323 (2009): 586-587. Supporting online materials at: www. sciencemag.org/cgi/content/full/323/5914/586/DC1.
5. Bao et al., "Learning and Scientific Reasoning," 587.
6. National Research Council, Inquiry and the National Science Education Standards: A Guide for Teaching and Learning (Washington, DC: National Academy Press, 2000)
7. Paul R. Gross et al., The State of State Science Standards (Washington, DC: Thomas B. Fordham Institute, 2005), www. edexcellence.net/doc/Science\ Standards.FinalFinal.pdf.
8. Science Framework for the 2009 National Assessment of Educational Progress (Washington, DC: National Assessment Governing Board, 2009).
9. See this interesting use of the FCl in connection with reasoning ability, scientific and in general: Vincent P. Coletta and Jeffrey A. Phillips, "Interpreting FCI Scores: Normalized Gain, Preinstruction Scores, and Scientific Reasoning Ability," American Journal of Physics 73, no. 12 (2005): 1172-1182. This paper includes a copy of the Lawson test of scientific reasoning.
10. Politically correct locution.
11. A recent, accessible treatment of these issues, with sufficient references to the literature of cognitive science, is: Daniel T. Willingham, Why Don't Students Like School?: A Cognitive Scientist Answers Questions about How the Mind Works and What It Means for the Classroom (San Francisco: Jossey-Bass, 2009).
12. Coletta and Phillips, "Interpreting FCI Scores," 1177.

## Just For You!

 These and Several Hundred others. For Friends and Family too! 1-800-774-9162
## www.buymags.com/aft

Alfred Hitchcock's Mystery $55.90 \quad 29.97$
Allure
American Baby American Civil War American History American Photo 21.0010 .50 American Woodworker $24.98 \quad 19.88$ Archaeology Architectural Digest Arthritis Today The Atlantic Automobile Backpacker Baseball Digest Better Homes 1 yr 22.9514 .94 \& Gardens $\quad 2$ yrs $44.00 \quad 22.00$ Bird Talk $\quad 27.9713 .99$ Black Enterprise $\quad 22.0017 .95$ Bon Appetit 28.00 24.00 *

BusinessWeek [50 iss]59.97 46.00 *
Car \& Driver
Cat Fancy 22.0010 .97 27.9715 .00


Civil War Times $\quad 39.95 \quad \mathbf{2 5 . 0 0}$
Coastal Living $\quad 36.0016 .00$

College Bound $\quad 59.0049 .00$
Conde Nast Traveler $19.97 \quad 12.00$ *
Consumer Reports $\quad 29.0029 .00$
Cookbook Digest $\quad 36.00 \quad 30.00$
Cooking Light $\left.\begin{array}{llll}1 \mathrm{yr} & 21.82 & 20.00 \\ 2 \mathrm{yrs} & 36.00 & \mathbf{3 0 . 0 0}\end{array}\right]$
Cosmopolitan $\quad 29.97$ 18.00 *
Country Living
Dell Horoscope Details
Discover 24.0012 .00 52.8729 .97 $15.00 \quad 7.97$ 29.9524 .95 Disney's Princess (age 4+) 49.5039 .97 Dog Fancy 27.9712 .00

Ebony
$1 \mathrm{yr} \quad 20.00 \quad 14.97$
2 yrs $40.00 \quad 24.97$

[^19]| Economist | 127.00 | $\mathbf{7 7 . 0 0}$ * |
| :--- | ---: | ---: |
| Elle | 28.00 | $\mathbf{1 4 . 0 0}$ |
| Elle Decor | 29.00 | $\mathbf{1 4 . 5 0}$ |
| Entertainment Weekly | 39.95 | $\mathbf{3 4 . 9 5}$ * |
| ESPN | 26.00 | $\mathbf{1 3 . 0 0}$ |
| Esquire | 15.94 | $\mathbf{8 . 9 7}$ |
| Essence | 22.00 | $\mathbf{1 8 . 9 6}$ |
| Every Day with Rachael Ray | 34.00 | $\mathbf{2 4 . 0 0}$ |
| Family Circle | 19.98 | $\mathbf{1 5 . 0 0}$ |


| The latest NEW YORKER <br> ideas,  <br>   <br> current  <br> issues  |  |
| :--- | :---: |
| A best buy $\$ 29.95$ |  |

## Family Fun

 $\begin{array}{lll}16.95 & 24.00 & 15.00\end{array}$ Fast Company [15 iss] 29.9419 .75 Field \& Stream $\quad 19.9710 .00$ Fitness $\quad 19.98 \quad 14.97$ FlyingFood \& Wine $\begin{array}{ll}19.98 & 14.97 \\ 26.00 & 13.00\end{array}$

Forbes
Foreign Affairs
Foreign Policy
Fortune [25 issues]
Girls Life
Glamour
Golf
Golf Digest
Golf World
Good
Housekeeping 2 yrs 39.9424 .00
GQ
Harper's Bazaar
Harper's Magazine Herb Companion Highlights For Children $34.95 \quad 29.64$ House Beautiful $\quad 19.97 \quad 12.00$ Instructor (K-8) InStyle [12 issues] Jet 38.0026 .00 The Kiplinger Letter $79.00 \quad 48.00$ Kiplinger's Personal Finance $23.95 \quad 12.00$ Kiplinger's Retirement Report $59.95 \quad 29.95$ Kitplanes Ladies Home Journal $16.97 \quad 9.99$ Latina Logic Lover's Logic Problems $29.94 \quad 19.97$ Lucky Macworld Mad Magazine Marie Claire www.buymags.com/aft

Publication Martha Stewart Living Men's Fitness Men's Journal Metropolitan Home Midwest Living Military History Modern Bride Money [12 issues]

Usual Your Price Price
39.8919 .95
NOW!
Vanity Fair
at a
special
member
rate.

| 1 year $\$ 15.00$ |
| :--- |
| 2 |

SAVE on 2 years $\$ 25.00$
More Magazine $\quad 18.00 \quad 14.97$
e Magazine
Mother Earth News
Mother Jones
Motor Trend
Motorboating
The Nation
Natural Health Natural History
$\begin{array}{lllll}\text { New York } & 1 \mathrm{yr} & 30.00 & 15.00 \\ 24.97 & 19.97\end{array}$
2 yrs 55.0237 .97
Old House Journal
Organic Gardening
Outdoor Life
19.9710 .00

Outdoor Photographer 19.9410 .98
$\begin{array}{lll}\text { Paper Magazine } & 14.95 \quad 12.97\end{array}$
Parenting $\quad 1$ yr $17.97 \quad 9.97$
Early Years $\quad 2$ yrs $35.94 \quad 14.97$
$\begin{array}{lllll}\text { Parenting } & 1 \text { yr } & 17.97 & 9.97\end{array}$
$\begin{array}{lllll}\text { School Years } & 2 \text { yrs } & 35.94 & 14.97\end{array}$
Parents
PC World $\quad 29.90 \quad 19.97$
Popular Photography $24.00 \quad 11.97$
Popular Science
$\begin{array}{ll}4.00 & 11.97 \\ 9.95 & 12.00\end{array}$

## - Best Titles <br> - LOWEST Rates - GREAT GIFTS!

 18.00 $18.00-13.97$ 18.0010 .00 18.0010 .00 23.9614 .00 52.0026 .00 31.0031 .00 $52.00 \quad 29.95$ 7.0013 .97 a member favorite!

Full year - JUST $\$ 77.00$
$\begin{array}{lll}\text { Publication } & \begin{array}{l}\text { Usual } \\ \text { Price }\end{array} & \text { Your } \\ & \text { Price }\end{array}$

ShopSmart
Slam [NBA]
Smart Money
Smithsonian
Sound \& Vision Southern Living Sporting News [1 year] 59.9020 .00 Sports Illustrated 89.0439 .95 The Weekly Standard 79.9647 .96 Stereophile 19.9412 .97

Sunset $\quad 24.0019 .95$ Technology \& Learning $48.00 \quad 29.95$
Teen Trend $\quad 16.0012 .95$
Teen Vogue $\quad 15.00 \quad 10.00$
18.0012 .00
This Old $\quad 1 \mathrm{yr} \quad 20.00 \quad 20.00$

House $\quad 2$ yrs $40.00 \quad 30.00$ Time [56 issues] 59.95 29.95 * Time Out - New York $39.94 \quad 19.97$
Town \& Country 28.0015 .00


## Extended Office Hours Mon.-Thur. 9am-7pm \& Fri. til 5pm ET

25.0022 .95 19.9712 .97 24.0015 .00 34.0012 .00 29.0012 .00 6.0028 .00

$\begin{array}{lll}\text { Travel \& Leisure } & 45.00 \quad 19.00\end{array}$
TV Guide
Unique Homes US Weekly
Vanity Fair
$1 \mathrm{yr} \quad 24.00 \quad 15.00$ 2 yrs 48.0025 .00 Vegetarian Times $\quad 19.9519 .95$

## Vogue

W Magazine
Weight Watchers
Wine Enthusiast Wired
Woman's Day
Women's Health
Workbench
Working Mother
World War II
Yoga Journal 1 yr
Yoga Journal 2 yrs $43.90 \quad 26.95$
Hundreds of Others Just Ask! $58.14 \quad 39.96$ $31.00 \quad 25.97$ 5.0052 .00 $\begin{array}{ll}19.95 & 19.95 \\ 29.95 & 21.00\end{array}$ 29.9014 .95 $17.70 \quad 14.95$ $35.00 \quad 26.95$ $24.00 \quad 12.00$ $18.00 \quad 9.99$ 17.97 14.97 * 22.0018 .00 $12.97 \quad 9.97$ 3.9525 .00
17.9714 .97 19.9512 .00 21.9714 .97
$32.70 \quad 26.97$ 21.9716 .94
$1 \mathrm{yr} \quad 21.00 \quad 15.97$ $42.00 \quad 20.97$ 24.9813 .96 29.9621 .95 28.9524 .00 17.9712 .00 27.9714 .99 25.9414 .95 24.0020 .00 24.0016 .00 29.9519 .95 $24.95 \quad 20.00$ 34.9724 .97 ,

Preschool Playroom
Prevention
Psychology Today 2 yr
Reader's Digest
large print edition
Real Simple [12 issues]
Redbook
Reptiles
Road \& Track
Rolling Stone
Runner's World
Sailing World
Saveur
Science Illustrated Scientific American Self
Seventeen
Shape

| Publication Name | Years | Price |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  | Total |  |

- Check enclosed payable to: AFTSS
- Charge to my credit card
$\square$ Visa MasterCard Discover $\square$ Amex
Acct: $\qquad$ Exp.


## Why should educators care about health insurance reform?


[^0]:    Editors' reply:
    A handful of readers wrote to express their disapproval of our choice to use the nonstandard spelling of all right (see

[^1]:    *There have been calls for math teachers in the education literature, among them the National Research Council's Adding It Up (see pages 397-398, available at www.nap. edu/catalog.php?record_id=9822\#toc) and the National Mathematics Advisory Panel's Foundations for Success (see Recommendation 20 on page xxii, available at www.ed. gov/about/bdscomm/list/mathpanel/report/final-report.pdf).

[^2]:    *An example of partitive division is to put 24 items in 6 bags (each with an equal number of items), and find that each bag has 4 items.

[^3]:    ${ }^{\dagger}$ This definitely would be appropriate for fifth-graders once the idea of using symbols for abbreviations is introduced and many examples are given for illustration.

[^4]:    *Very large numbers are already an abstraction to children, but children tend not to be systematically exposed to such numbers the way they are to fractions.
    ${ }^{\dagger}$ Rational numbers consist of fractions and negative fractions, which of course include whole numbers.
    *See, for example, page 4-40 of the National Mathematics Advisory Panel's "Report of the Task Group on Learning Processes," www.ed.gov/about/bdscomm/list/math panel/report/learning-processes.pdf.
    ${ }^{\text {§ }}$ We exclude complex numbers from this discussion, as they are not appropriate for the elementary grades.

[^5]:    **Naturally, the theorem on equivalent fractions implies that $10 / 24=5 / 12$, as $10 / 24=(2 \times 5) /(2 \times 12)$, but contrary to common belief, the simplification is of no great importance. Notice in particular that there was never any mention of the "least common denominator."

[^6]:    *This term is historically correct in the sense that the Hindu-Arabic numeral system was transmitted to the West from the Islamic Empire around the 12th century, and the Arabs themselves got it from the Hindus around the 8th century. However, recent research suggests a strong possibility that the Hindus, in turn, got it from the Chinese, who have had a decimal place-value system since time immemorial. See Lay Yong Lam and Tian Se Ang, Fleeting Footsteps: Tracing the Conception of Arithmetic and Algebra in Ancient China (Hackensack, NJ: World Scientific, 1992).
    ${ }^{\dagger}$ Historically, 0 was not among the symbols used. The emergence of 0 (around the 9th century and beyond) is too complicated to recount here.

[^7]:    *The treatment of fraction multiplication in textbooks and in the education literature is mostly defective, but one can consult pages 62-74 of http://math.berkeley.edu/~wu /EMI2a.pdf for an introduction.

[^8]:    *See, for example, the National Mathematics Advisory Panel's "Report of the Task Group on Conceptual Knowledge and Skills," especially Appendix B, www.ed.gov/ about/bdscomm/list/mathpanel/report/conceptual-knowledge.pdf, and "Report of the Subcommittee on Instructional Materials," www.ed.gov/about/bdscomm/list/ mathpanel/report/instructional-materials.pdf.
    †'See, for example, the National Council on Teacher Quality's No Common Denominator, www.nctq.org/p/publications/reports.jsp.
    *See Recommendation 7 on page xviii and Recommendations 17 and 19 on page xxi in Foundations for Success: The Final Report of the National Mathematics Advisory Panel, www.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf.
    ${ }^{\text {§ }}$ Foundations for Success, Recommendation 20, page xxii, see note above for URL.

[^9]:    Karin Chenoweth, a senior writer with the Education Trust, is the author of How It's Being Done: Urgent Lessons from Unexpected Schools (Harvard Education Press, 2009), from which this article is excerpted, and It's Being Done: Academic Success in Unexpected Schools (Harvard Education Press, 2007). From 1999 to 2004, she was an education columnist for the Washington Post, and before that was the senior writer and executive editor of Black Issues in Higher Education (now Diverse). For more information on How It's Being Done, visit www.hepg.org/hep/book /102/HowItsBeingDone.

[^10]:    *All of the schools mentioned in this article are profiled in either my new book, How It's Being Done: Urgent Lessons from Unexpected Schools (Harvard Education Press, 2009), or in my 2007 book, It's Being Done: Academic Success in Unexpected Schools (Harvard Education Press).

[^11]:    Jennifer Dubin is the assistant editor of American Educator. Previously, she was a journalist with the Chronicle of Higher Education.

[^12]:    *Patsy Wang-Iverson, "What Makes Lesson Study Unique?" in Building Our Understanding of Lesson Study, ed. Patsy Wang-Iverson and Makoto Yoshida (Philadelphia: Research for Better Schools, 2005), 19.

[^13]:    *To learn more about the study, see "Teaching Is a Cultural Activity," American Educator 22, no. 4 (Winter 1998), available at www.aft.org/pubs-reports/american_ educator/winter98/TeachingWinter98.pdf. The official report of the study is available at http://nces.ed.gov/pubs99/1999074.pdf.
    ${ }^{\dagger}$ The VTO is jointly affiliated with two national unions: the American Federation of Teachers and the National Education Association. VTO President Andrew Spar has supported teachers' participation in lesson study, providing funding for some members to attend national lesson study conferences to improve their practice.

[^14]:    ${ }^{\ddagger}$ In December 2008, interested teachers attended an organizational meeting where they split into a math group and a writing group. An initial meeting for lesson study usually takes place in September. But Pine Trail held it a few months later because teachers had to learn a new system for state testing, which cut into their time.

[^15]:    Paul R. Gross is University Professor of Life Sciences, emeritus, and former vice president and provost, University of Virginia. He has taught science at New York University, Brown University, the Massachusetts Institute of Technology, and the University of Rochester, and is former director and president of the Marine Biological Laboratory, Woods Hole, Massachusetts. He has written numerous articles and books on topics ranging from molecular biology of development to the intersection of science and culture.

[^16]:    *In the article, scientific reasoning is not simply subsumed under content; the authors' use of "content" implies that, for them, the word means something like just the facts, ma'am-with perhaps some very ad hoc concept juggling and problem solving.

[^17]:    *Numbers of test takers in all cases were large enough for there to be no doubt that the calculated means are properly representative.

[^18]:    ${ }^{\dagger}$ The possibility that this result was due to a ceiling effect occurs immediately to any test-hardened teacher, and these energetic authors did not fail to consider it. Online supplementary material includes their independent investigation of that possibility, with the result that no ceiling effect is a likely cause of the near-identical, 74 percent mean scores for Chinese and American college freshmen who are preparing to major in science or engineering.

[^19]:    * These rates for AFT members and college students only.

