Mathematics for Teaching: Then and Now

In light of the current debate over how much and what kinds of mathematical knowledge teachers need, we thought it might be interesting to see how these questions were answered in years past. The problems below are from the State of California's teacher certification exam of December 1874. In addition to the three mathematics portions of the test, prospective teachers were examined in the areas of written grammar, oral grammar, geography, U.S. history, the theory and practice of teaching, physiology, natural philosophy, the constitutions of the United States and California, California school law, penmanship, natural history, composition, reading, orthography, defining, vocal music, and industrial drawing.

—EDITOR

ARITHMETIC

1. Divide 8,786,742 by the factors 7, 5, and 2. Explain the principle of obtaining the true remainder.

2. What is the greatest common divisor and the least common multiple of the numbers 18, 36, and 24? Explain the principle of obtaining each.

3. What is the difference between 2 miles, 5 furlongs, 6 rods, 3 yards, 2 feet, 7 inches, and 7 furlongs, 39 rods, 4 yards, 2 feet, 8 inches? Prove your work by subtraction in decimals.

4. Write in words and analyze the following fractions: ¾, 2⅓, .3, and .00007.

5. If ⅔ of 7 tons of coal cost $93⅓, what will ¾ of 5 tons cost? Work by analysis and prove by proportion.

6. What will it cost to build a wall 650 feet long, 8 feet high, and 2⅔ feet thick, at $9.75 per 1,000 bricks—each brick being 8 inches long, 4 inches wide, and 2 inches thick?

Excerpted from the Sixth Biennial Report of the Superintendent of Public Instruction of the State of California, School Years 1874 and 1875, *published in 1875 by G.H. Springer, Sacramento, California.*

7. A grocer sold 4 barrels of sugar for $30 each; on two barrels he gained 20 percent, on the other two barrels he lost 20 percent; did he gain or lose on the whole?

8. A man received $16.12½ interest on a sum of money, which had been loaned 3⅓ years at 9 percent. What was the principal?

9. A flagstaff 75 feet high stands in the center of a square lot containing two acres; what is the length of a rope extending from one corner to the top of the staff?

10. Extract the cube root of $\frac{9}{21\frac{1}{3}}$

11. What are the names of the units used in the metric system for length, surface, solid, capacity, and weight?

MENTAL ARITHMETIC

1. Paid $2.50 for 5 yards of ribbon, at 12½ cents per yard, and 3 books at 37½ cents each. How much change did I receive back?

2. What percent of 60 is 12?

3. How many men can perform the same amount of work in 12 days that 6 men can in 4 days?

4. If a man travels 1 mile in 20 minutes, how many hours and minutes will it take him to travel 17 miles?

5. If A and B can do a piece of work in 4 days, and A can do it alone in 6 days, how long would it take B to do it?

6. A man being asked how many sheep he had, replied that if he had one and one half times as many more, and 2½ sheep, he would have fifty. How many had he?

7. A man after spending one half of his money and one third of the remainder, had $10 left. How much had he at first?

8. How many car tickets, at 6¼ cents each, can be purchased for $7.50?

9. How long will it take a man to clear $100 if he gets $1.50 a day, and pays $4 a week for board?

10. If you get 2½ credits each for 8 questions out of 10, what will be your percentage?

ALGEBRA

1. What is a reciprocal? Zero power? Negative exponent?

2. Find the prime factors of $6x^2 + xy - 9x - y^2 + 3y$.

3. Divide a by $\frac{x}{x+y} \times \frac{a}{x-y}$

4. What is the greatest common divisor and the least common multiple of $x^2 - 4a^2$, $(x+2a)^3$, and $(x-2a)^3$?

5. If B gives A $5 of his money, A will have twice as much as B has left; but if A gives B $5, then A will have but three fourths as much as B will have. How much has each?

6. Extract the square root of $a^4 - a^3b + \frac{3a^2b^2}{4} - \frac{ab^3}{4} + \frac{b^4}{16}$

7. A merchant sold a piece of cloth for $39, and gained as much percent as it cost him. How much did it cost him?

8. Divide $7y^2$ by $\frac{10ax}{3by}$

9. Two houses standing on opposite sides of a street 84 feet in width, are respectively 67 and 54 feet in height. What length of rope will reach from the top of one house to the top of the other?

Note: 1 mile = 5,280 feet or 1,760 yards or 8 furlongs; 1 furlong = 660 feet or 220 yards or 40 rods; 1 rod = 16.5 feet or 5.5 yards; 1 acre = 43,560 square feet or 4,840 square yards or 160 square rods.