

A Video Series on Mathematical Reasoning

The Common Core State Standards for Mathematical Practice require students to learn mathematics content, as well as to reason quantitatively and abstractly, to construct viable arguments, and to critique the reasoning of others. Working with Teaching Channel, AFT math teachers have tried to capture how students grow in these areas throughout their school careers. To that end, they have created “Mathematical Reasoning through the Grades,” a video series that records the dramatic changes that take place in student understanding of mathematical concepts—from a kindergartner’s first attempts to connect bits of information, to an intermediate student’s wrestling with fractions, to a high school student’s application of trigonometry in understanding how to build and fly drones. The video series begins in kindergarten with an explanation of numbers and ends in grade 11 with trigonometry. Videos and corresponding lesson plans with handouts are available for free at www.bit.ly/1bvt5g1. The following are moments from each lesson that demonstrate mathematical reasoning.



ELEMENTARY VIDEOS

First-Grade Lesson: Leprechaun Traps

In this video, a first-grader identifies a mystery number located in the third row and third column of a 100-square grid as 33. He is then asked how many numbers to the next “friendly” number (a multiple of 10), and he identifies the friendly number as 40 “because it goes 3, 4.” His answer shows he is building on his knowledge of the structure of the number system. The teacher scaffolds the language and also clarifies this student’s thinking of 3 tens and 4 tens for other children. www.bit.ly/1hiXOyY

Fourth-Grade Lesson: Multiplying Whole Numbers and Fractions

In the intermediate grades, it is much easier to identify students’ reasoning. Children now have a better vocabulary and more mathematical knowledge. They are beginning to use words such as “because” and “since” and “so” to help them explain why they have drawn certain conclusions. In this lesson, fourth-graders demonstrate their knowledge of yards and fractions. www.bit.ly/1jFlv4U

Fifth-Grade Lesson: A Passion for Fractions

In fifth grade, students multiply two fractions with different denominators, a more complex operation than multiplying a whole number and a fraction. Their reasoning revolves around representations of a situational problem. They must explain which representations are correct, and which are incorrect. www.bit.ly/1g4G7hl

MIDDLE AND HIGH SCHOOL VIDEOS

Eighth-Grade Lesson: Conjecturing about Functions

This lesson enables students to look at sets of functions and draw conclusions that hold true for all functions of that type. The video captures the first day of the lesson, during which students are making observations that will help them make conjectures.

As the video shows clearly, students are beginning to extend their thinking—with the help of targeted questions the teacher poses—beyond the specific problem they are solving. www.bit.ly/1jFhIKh

Eleventh-Grade Lesson:

Sine and Cosine—Trigonometry in Flight

In this class, perhaps the most commonly used word to help students learn how to build quadcopters (multicopters propelled by four rotors) is “why.” Why must the propellers turn in opposite directions? Why is amplitude the constant in all the graphs? Why calculate the longest distance? Why not just use the Pythagorean theorem? When a student displays a graph that depicts a quadcopter’s left turn, and explains that its right propeller is going fast (high frequency) and then slows down to match the frequency of the left

propeller, the teacher asks him why. All answers to the teacher’s questions are content-based and involve students justifying their answers. www.bit.ly/1ov6jHY



SOURCE: SHARE MY LESSON, FOURTH-GRADE LESSON: MULTIPLYING WHOLE NUMBERS AND FRACTIONS, WWW.BIT.LY/1JFLV4U.

THE TEACHER’S ROLE

In each instance, teachers prod and probe to help students better articulate what is important mathematically. Teachers help them recognize the underlying concepts that are broader than particular problems or examples. Mrs. Wright, the first-grade teacher, reiterates much of

what children say, and provides specific models from which they can learn. Mrs. Spies and Mrs. Pittard provide examples for their fourth- and fifth-graders to consider and discuss. They also offer clues within questions to help students along the way.

In her lesson with eighth-graders, Mrs. McPhillips encourages the use of color and precisely defined variables with labels to help students recognize key patterns and make their justifications clear to others. Eleventh-grade teachers Mrs. Brookins and Mr. James push students to connect mathematics and physics principles to an ambitious long-term, real-world task. At every level, these teachers use the guidance provided by the Common Core State Standards for Mathematical Practice to direct their questions to the content students are learning and to the usefulness of mathematics in real life.

—FROM THE AFT’S EDUCATIONAL ISSUES DEPARTMENT