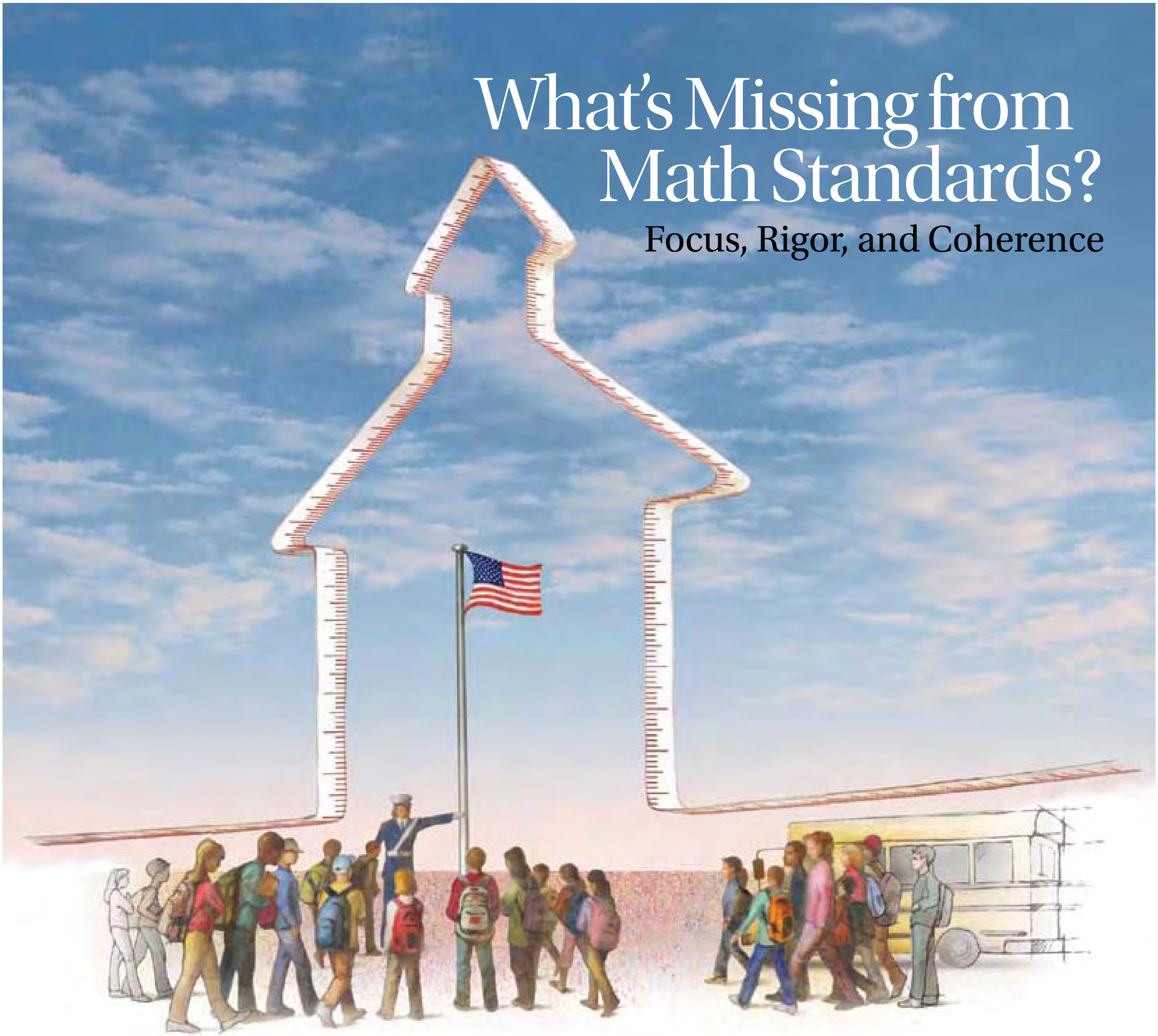
What's Missing from Math Standards?

Focus, Rigor, and Coherence



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By William H. Schmidt

Why do some countries, like Singapore, Korea, and the Czech Republic, do so much better than the United States in math? I've heard all sorts of reasons; diversity and poverty top the list. But after some 15 years conducting international research, I am convinced that it's the diversity and poverty of U.S. math standards—not the diversity and poverty of U.S. students—that are to blame.

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The single most important result of the Third International Mathematics and Science Study (TIMSS) is that we now know that student performance is directly related to the nature of the curricular expectations. I do not mean the instructional practices. I mean the nature of what it is that children are to learn within schools. (In the U.S., the curricular expectations are usually referred to as standards; in other countries they are known by various names.) After all, what is more central to schooling than those things we, as a society, have chosen to pass on to our children?

The TIMSS research has revealed that there are three aspects of math expectations, or standards, that are really important: focus, rigor, and coherence. Let's take a brief look at each.

Focus is the most straightforward. Standards need to focus on a small enough number of topics so that teachers can spend months, not days, on them. I'll just give you one illustration: in the early grades, top-achieving countries usually cover about four to six topics related to basic numeracy, measurement, and arithmetic operations. That's all. In contrast, in the U.S., state and district standards, as well as textbooks, often cram 20 topics into the first and second grades. That's much more than any child could possibly absorb.

Rigor is also pretty straightforward—and we don't have enough of it. For example, in the middle grades, the rest of the world is teaching algebra and geometry. The U.S. is still, for most children, teaching arithmetic. It's not rocket science: other countries outperform us in the middle and upper grades because their curricular expectations are so much more demanding, so much more rigorous.

Coherence is not quite as easy to grasp, but I believe it is the most important element. Coherent standards follow the structure of the discipline being taught. All school subject matter derives from some academic discipline, be it geography, history, mathematics, physics, etc. Once that formal academic body of knowledge has been parsed out and sequenced from kindergarten through 12th grade, it should reflect the internal logic of the discipline. This is especially important in mathematics, which is very hierarchical. Topics in math really need to flow in a certain logical sequence in order to have coherent instruction. If you look at the math curricula of top-achieving countries, you see a very logical sequence (which I describe in the box on p. 24). The more advanced topics are not covered in the early grades. Now, that seems obvious—until you look at state and district standards in the U.S. Everything is covered everywhere. Far from coherent, typical math standards in the U.S. often appear arbitrary, like a laundry list of topics.

And it shows in our abysmal math achievement. On the math portion of the 2003 Trends in International Mathematics and Science Study, just seven percent of fourth- and eighth-graders in the U.S. attained the advanced level; in comparison, in Singapore (the top achieving nation), 38 percent of fourth-graders and 45 percent of eighth-graders attained the advanced level (Gonzales et al., 2004).

So it's important to ask, why do we have such unfocused, undemanding, and incoherent math standards? I attribute it to the long tradition in the U.S. of shared responsibility in curriculum decision-making, as well as a complex decentralized arrangement for schooling and curriculum development. What many other countries take for granted is problematic, and political, in the U.S.

The development of standards, even at a very localized level, does not occur in a vacuum. Inevitably, the process is influenced by standards from other organizations, such as districts, states, and national associations. It is also influenced by examination of textbooks and standardized tests, as well as an intuitive sense of what is currently being taught in the classroom. The resulting multiple possibilities, coupled with the U.S. notion of individualism and the virtual absence of input from the academy (i.e., university professors and research mathematicians), make defining the sequence of topics an exercise in democratic consensus-making. Unfortunately, standards setting in the U.S. is more conducive to politically motivated, *ad hoc* approaches to content than to discipline-based ones (Schmidt et al., 2005).

Perhaps that explains why several states, instead of addressing the lack of coherence, focus, and rigor in their standards, have tried to raise math achievement by increasing Carnegie units required for high school graduation and specifying higher-level courses that must be completed (Dounay, 2006). Unfortunately, this strategy won't work. Neither seat time nor credentials are reasonable indicators of student learning.

One researcher recently reported that despite having taken Geometry and Algebra II, 60 percent of low-income, 65 percent of African-American, and 57 percent of Hispanic students in Texas failed the state test that covered Algebra I. Here's how the researcher summed up the situation: "While truth-in-labeling practices in the food industry ensure that orange drink cannot be labeled orange juice without legal ramifications, schools have no such safeguards in place. Algebra I can be placed on any child's transcript without any guarantee about the content taught or learned" (Rutherford, 2005, as cited in Dounay, 2006).

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A study my colleagues and I recently conducted with about 6,000 students from eight high schools in two districts had similar findings (Kher et al., 2007). When we surveyed teachers as to what was actually taught, we found great variability among courses with the same title. In addition, we were quite surprised at just how many math courses were being offered. While one of the districts offered 20 math courses, the other offered 68—including seven varieties of Algebra 1 that ranged from Fundamentals of Algebra to Basic Algebra to Algebra 1A.*

The courses students take to fulfill graduation requirements clearly affect what they learn and their future academic options. But with this kind of variability in course offerings, how can high school students find the rigorous courses they need? Lack of clear standards coupled with a smorgasbord of choices creates a set of artificial tracks in the curriculum that adversely affect mathematical literacy, and also limit students' future educational and career opportunities. The analyses of TIMSS data show strong relationships cross-nationally between content standards and both what teachers teach and what students learn (Schmidt et al., 2001). Curricular expectations in high-performing countries focus on fewer topics, but also communicate the expectation that those topics will be taught in a deeper, more profound way. This is not happenstance; it means making real choices about what to teach and, of equal importance,

*As if that weren't bad enough, the district also offered Life Math, Consumer Math, Basic Math, etc.

articulating those choices in a consistent manner in key instructional supports like standards, textbooks, and assessments (Newman et al., 2001).

I've been beating the drum for focus, rigor, and coherence for many years, and there has been some progress. Some of the more recent standards are more focused, but they're still not very coherent. Many states have reduced the number of topics per grade, but sometimes they have removed the wrong topics, making their standards even more incoherent. In order for U.S. math standards to improve, states and districts must bring mathematicians into the standards setting process—and push the politics out.

Better still, states and districts should work together to establish national (if not federal) math standards. More than 30 states have joined forces through Achieve, Inc., so the U.S. may already be headed toward *de facto* national math standards. Along the way, looking to other countries would serve us well. The vast majority of the 40-plus countries participating in TIMSS had common national standards for all K-8 students. Even in countries with different schools for different types of students, the grade-level curricular expectations were usually the same. To many people in the U.S., common national standards are synonymous with federal standards. But "national" does not have to mean "federally imposed." TIMSS showed that the final decision regarding specific aspects of curriculum and its implementation varied greatly among countries, even when a common set of national content standards guided education overall. ☐

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World-Class Math Standards

To find out what world-class math standards in grades 1-8 would look like, we created a composite of the top-achieving countries' math curricula.* What we found was a three-tier pattern of increasing mathematical complexity. The first tier, covered in grades 1-5, includes an emphasis primarily on arithmetic, including whole-number concepts and computation, common and decimal fractions, and estimation and rounding. The third tier, covered in grades 7 and 8, consists primarily of advanced number topics, including exponents, roots, radicals, orders of magnitude, and the properties of rational numbers, algebra, including functions and slope, and geometry, including congruence and similarity and 3-dimensional geometry. Grades 5 and 6 appear to serve as an overlapping transition or middle tier marked by continuing attention to arithmetic topics (especially fractions, decimals, estimation, and rounding), but with an introduction to the topics of percentages, negative numbers, integers and their properties, proportional concepts and problems, 2-dimensional coordinate geometry, and geometric transformations, all of which, except for percentages, were also topics found in the third stage. Thus, grades 5 and 6 serve as a point of transition where attention to topics such as proportionality and coordinate geometry led to the formal treatment of algebra and geometry that is characteristic of the third stage.

The implied curriculum structure also includes six topics that provide a form of continuity across all three stages. These topics—measurement units; perimeter, area, and volume; algebraic equations, including the representation of numerical situations and the informal solution of simple equations; data representation and analysis; and basic two-dimensional geometry including points, lines, angles, polygons and circles—appear to ensure stability across the three tiers, serving as buttresses supporting the overall curriculum structure. Those buttresses include the fundamentals of algebra, geometry, measurement and data analysis, and, by way of the implied breadth of these topics, could move from their most elementary aspects to the beginnings of complex mathematics.

When we examined state and district math standards, the contrast with the international composite was striking. Not only is the organizing principle underlying these standards unlike that of the top-achieving countries, it actually seems illogical. The organizing principle (if one can call it that) seems to include almost every topic at almost every grade.

For a much more in-depth look at both the international composite and U.S. math standards, see the Summer 2002 issue of *American Educator*, available online at **www.aft.org/pubs-reports/american_educator/summer2002/curriculum.pdf**.

—W.H.S.

* We studied the curricular expectations of Singapore, Korea, Japan, Hong Kong, Belgium (Flemish), and the Czech Republic; these were the top-achieving countries on the TIMSS math assessment of seventh- and eighth-graders.